Generalized Liquid-Burning Analysis

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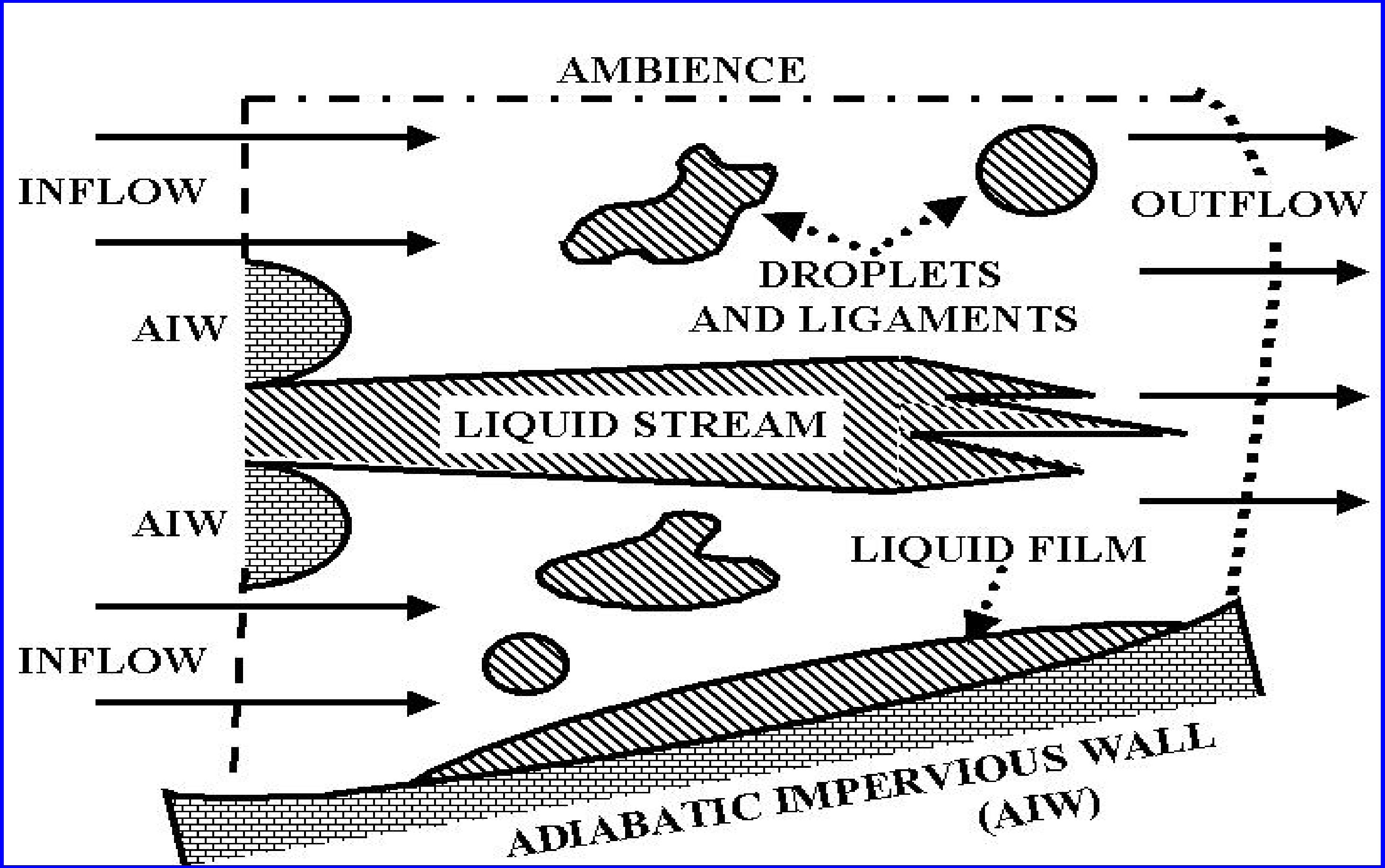


Liquid jets, wall films, pools, ligaments, and droplets

Major Assumptions

- **Fourier heat conduction**
- **Fickian mass diffusion, multicomponent flow**
- **Unitary Lewis number**
- **One-step oxidation kinetics**
- **Low Mach number : negligible kinetic energy; pressure gradients affect momentum balance but not energy balance**
- **Negligible radiation**
- **Allow unsteady viscous or inviscid flow with steady or quasi-steady pressure**
- **Phase-equilibrium law gives** $Y_{Fs}(T_s)$
- **No oxygen at liquid surface (diffusion flame)**

Basic Gas-Phase Equations

Species conservation

$$L\left(Y_i\right) \equiv \rho \frac{\partial Y_i}{\partial t} + \rho \boldsymbol{u} \cdot \nabla Y_i - \nabla \cdot \left(\rho D \nabla Y_i\right) = \rho \dot{w}_i \; ; \; i = F,\ O,\ P$$

Energy

$$L\left(h\right) \equiv \rho \frac{\partial h}{\partial t} + \rho \boldsymbol{u} \cdot \nabla h - \nabla \cdot \left(\rho D \nabla h\right) = -\rho \dot{w}_F Q + \frac{\partial p}{\partial t}$$

Sensible enthalpy definition

$$h = \sum_i Y_i \int_{T_{ref}}^{T} c_{pi}\left(T'\right) dT' = \sum_i Y_i h_i = \int_{T_{ref}}^{T} c_p\left(T'\right) dT'$$

$$c_p \nabla T = \nabla h - \sum_i h_i \nabla Y_i$$

Plus continuity and momentum equations

Conserved Scalar Formulation

Define Shvab-Zel'dovich (S-Z) variables

$$\alpha_1 = h + \nu Q Y_O \qquad \alpha_2 = Y_F - \nu Y_O$$

$$L(\alpha_i) \equiv \rho \frac{\partial \alpha_i}{\partial t} + \rho \boldsymbol{u} \cdot \nabla \alpha_i - \nabla \cdot (\rho D \nabla \alpha_i) = 0$$

- ***S-Z variables are conserved gas-phase scalars, neither created or destroyed by chemical reaction since no source term appears.***
- ***S-Z variables are not passive scalars; they can affect the velocity field. Also they can be affected by the velocity field.***

Gas-Phase Boundary Conditions

- ***Adiabatic, impervious walls:***

$$\frac{\partial \alpha_i}{\partial n} = 0\ ;\ i = 1,\ 2$$

- ***Inflow and/ or ambient boundaries, temporal variation is allowed:***

$$\alpha_i = \alpha_{i\infty}(t)\ ;\ i = 1,\ 2$$

- ***Outflow boundaries:***

$$\frac{\partial \alpha_i}{\partial n} = 0\ ;\ i = 1,\ 2$$

- ***Vaporizing (or condensing) liquid surfaces:***

$$\dot{m}\left(1 - Y_{Fs}\right) = -\rho D \left.\frac{\partial Y_F}{\partial n}\right)_s$$

$$L_{eff} = L + \dot{q}_\ell / \dot{m}$$

$$\left.\frac{\partial h}{\partial n}\right)_s = \frac{\dot{m} L_{eff}}{\rho D} = -\frac{L_{eff}}{1 - Y_{Fs}} \left.\frac{\partial Y_F}{\partial n}\right)_s$$

Super-Scalar Formulation

Consider L_{eff} / (1-$Y_{Fs}(T_s)$) to be steady or quasi-steady. Then a super scalar S can be constructed which is uniform over the gas field (but possibly time-varying).

$$S = \alpha_1 + \frac{L_{eff}}{1 - Y_{Fs}} \alpha_2 = h + \nu \left(Q - \frac{L_{eff}}{1 - Y_{Fs}} \right) Y_O + \frac{L_{eff}}{1 - Y_{Fs}} Y_F$$

$$S = S_\infty(t) = h_\infty + \nu \left(Q - \frac{L_{eff}}{1 - Y_{Fs}} \right) Y_{O\infty} + \frac{L_{eff} Y_{F\infty}}{1 - Y_{Fs}} = h_f$$

The limiting diffusion flame enthalpy and temperature can readily be determined.

Super-Scalar Formulation (continued)

$$h\left(T_f\right)=h\left(T_\infty\right)+\nu\left(Q-\frac{L_{eff}}{1-Y_{Fs}}\right)Y_{O\infty}+\frac{L_{eff}}{1-Y_{Fs}}Y_{F\infty}=h\left(T_s\right)+\frac{L_{eff}}{1-Y_{Fs}\left(T_s\right)}Y_{Fs}\left(T_s\right)$$

$$\frac{h_\infty-h_s+\nu QY_{O_\infty}}{L_{eff}}=\frac{Y_{F_S}-Y_{F_\infty}+\nu Y_{O_\infty}}{1-Y_{F_S}}\equiv B$$

- ➢ ***The Super Scalar is more than a conserved scalar; it is a uniform scalar of the Crocco-Busemann type.***
- ➢ ***The limiting flame temperature can be immediately calculated from liquid-surface and ambient / inflow conditions WITHOUT solving scalar or velocity field.***
- ➢ ***Recovers Spalding Transfer Relation; surface values are related to ambient / inflow conditions.***

Unsteady Liquid Phase

$$L_{eff} = L + \dot{q}_\ell / \dot{m}$$

For S to be uniform, L_{eff} must be uniform over the liquid interface but may vary with time in a quasi-steady fashion: i.e., at the same rate as gas-phase variables but slower than unsteady liquid-phase variables.

EXAMPLES

- Spherical symmetry
- Wet-bulb temperature

$$L_{eff} = L$$

- Thin liquid thermal layer

$$L_{eff} = L + c_\ell (T_s - T_O)$$

- Emmons boundary layer over vaporizing surface -- Heat and mass transfer vary over surface in proportion to each other

$$\dot{q}_\ell / \dot{m} = const.$$

$$Nu \equiv \frac{d \left. \frac{\partial h}{\partial n} \right)_s}{h_\infty - h_s + \nu Q Y_{O\infty}} = \frac{\dot{m} d / \rho D}{B}$$

Super-Scalar Characteristics

- ***Formulation and results are affected by geometrical configuration, velocity field, transport properties, chemical kinetic constants, and liquid-phase unsteadiness only through L_{eff}***
- ***S is not equal to total enthalpy h_t except at diffusion flame; $h_t > S$ on oxidizer side; $h_t < S$ on fuel side.***
- ***Mixture-fraction or S-Z variable analyses could be developed to yield the same results but are more complicated since they are spatially varying; note mixture-fraction or S-Z approach have not yet been applied to a general vaporizing liquid case.***
- ***It is required that $L_{eff} / (1 - Y_{Fs})$ is uniform over all liquid surfaces and steady or quasi-steady.***

Configuration Examples for Analysis

- ***Quasi-steady isolated droplet vaporization and burning***
- ***Unsteady isolated droplet vaporization and burning***
- ***Droplet array vaporization and burning***
- ***Group combustion***
- ***Vaporizing and burning wall films***
- ***Vaporizing and burning fuel pools***

Potential Benchmarking Examples

- ***Turbulent reacting flow models and calculations; non-fluctuating super scalar.***
- ***Non-unitary Lewis number calculations; verify analysis and / or code in the Le = 1 limit.***
- ***Non-uniform*** L_{eff} ***problems; verify analysis and/or code.***

Remarks

- *For Le = 1 and quasi-steady variations of pressure and L_{eff}, a certain scalar combination is uniform across the gas field.*
- *While the super-scalar is constructed in only a few analytical steps beyond published results, it provides capability for more direct and faster computations even for problems where S is not uniform.*
- *It allows limiting flame temperature calculations that were once complex in the simple configurations to be simple in the most complex configurations.*
- *The calculations of liquid-surface temperature and mass fraction are equivalent to Spalding Transfer Method.*
- *A useful approach to single-component and multi-component liquid-fuel burning problems has been identified.*
- *A useful method for benchmarking models and computations has been presented.*
- *Extension to gaseous diffusion flames is straightforward.*

New Solution Method for Gas Fiield

- A generalized method for solving quasi-steady liquid-fuel combustion problems
- Analysis applies to liquids with gas interfaces: droplets, sprays, films, pools, and streams
- Geometrical effects are separated from effects of gas-phase properties or boundary conditions
- Can be coupled with an unsteady liquid-phase
- Can be used for vaporization without combustion

New Assumptions

- Quasi-steady gas phase
- Temperature is spatially uniform over all liquid-gas interfaces
- Constant ambient temperature, pressure, and oxidizer mass fraction
- No forced or natural convection; Stefan flow only

Other Assumptions are retained, e.g.,

- One-step chemical reaction
- Unitary Lewis number (for cases with combustion)
- Negligible radiation
- Low Mach number
- No oxygen at liquid surface if flame exists

Governing Equations

Mass, energy, and species conservation,

$$\nabla \cdot (\rho \vec{V}) = 0$$

$$\nabla \cdot (\rho \vec{V} \alpha_i - \rho D \nabla \alpha_i) = 0$$

$$p = \rho R T = p_\infty$$

Coupling functions

$$\alpha_1 = h + \nu Q Y_O$$

$$\alpha_2 = Y_F - \nu Y_O$$

Problem Formulation

The mass flux is governed by a normalized potential function

$$\rho\vec{V} = \overline{\rho D}\ln(1+B)\nabla\Phi$$

$$B = \frac{h_\infty - h_S + \nu Q Y_{O\infty}}{L_{eff}} = \frac{\nu Y_{O\infty} + Y_{FS}}{1 - Y_{FS}}$$

$$L_{eff} = L + \frac{\dot{q}_l}{\left|\rho\vec{V}\right|_S}$$

Solutions to the species and energy equations have the form

$$\alpha_i = A_i + B_i(1+B)^\Phi$$

Problem Formulation

Enthalpies, composition, flame locations, and $\overline{\rho D}$ can be obtained for a fixed liquid-surface temperature

$$\overline{\rho D} = \overline{\lambda / c_p} = \left(\int_0^1 \frac{d\Phi}{\lambda / c_p} \right)^{-1}$$

$$(1+B)^{\overline{\rho D}\int_0^{\Phi} \frac{d\Phi'}{\lambda / c_p}} = 1 + \frac{h - h_S + \nu Q Y_O}{L_{eff}} = 1 + \frac{Y_F - Y_{FS} - \nu Y_O}{Y_{FS} - 1}$$

$$\overline{\rho D} \int_0^{\Phi_F} \frac{d\Phi'}{\lambda / c_p} = \frac{-\ln(1 - Y_{FS})}{\ln(1 + B)}$$

Problem Formulation

For any constant value of ρD , the equations are simplified and uncoupled. One choice is the average value

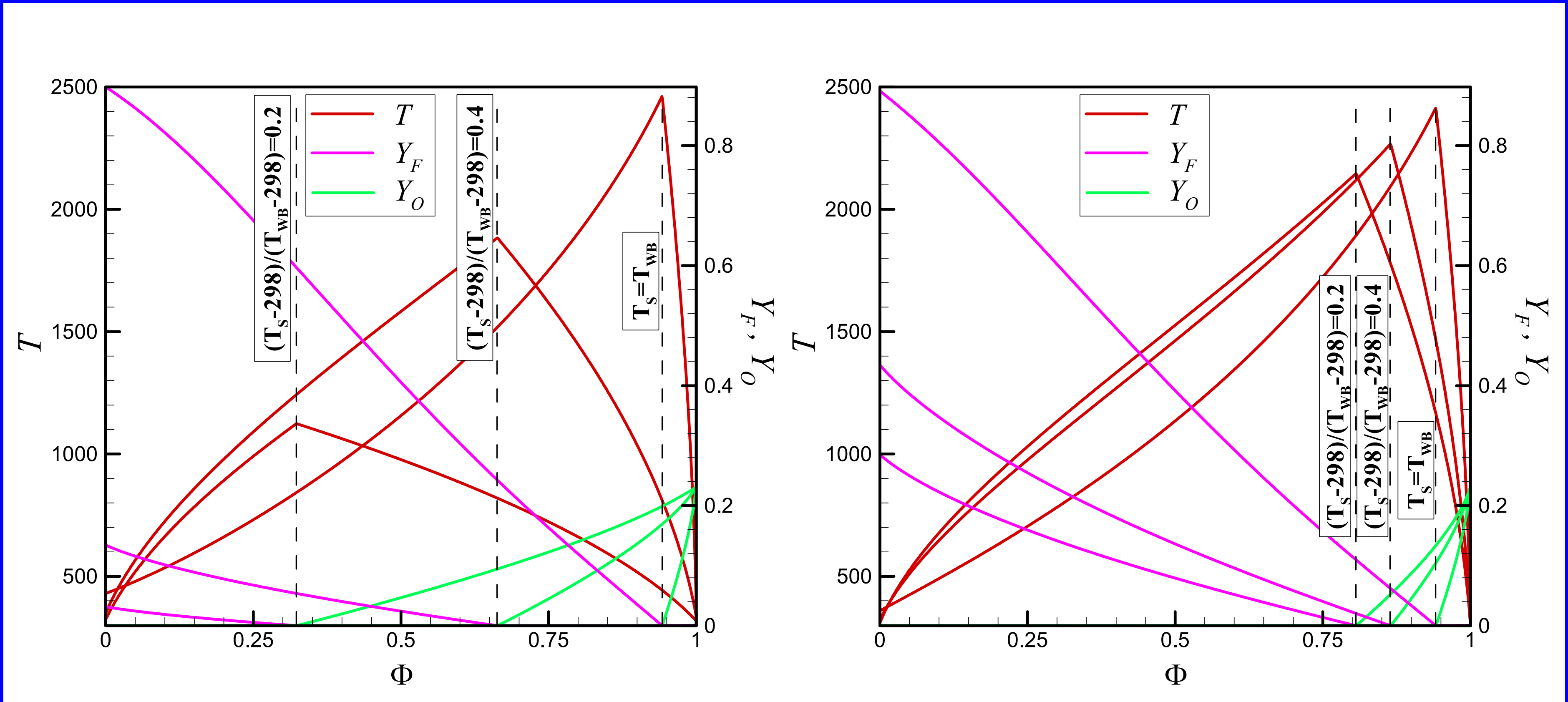
$$\overline{\rho D} = \overline{\lambda / c_p} = \left(\int_0^1 \frac{d\Phi}{\lambda / c_p} \right)^{-1}$$

$$(1+B)^{\Phi} - 1 = \frac{h - h_S + \nu Q Y_O}{L} = \frac{Y_F - Y_{FS} - \nu Y_O}{Y_{FS} - 1}$$

$$\Phi_F = \frac{-\ln(1 - Y_{FS})}{\ln(1+B)}$$

Mixture temperature, fuel and oxidizer mass fractions for decane and heptane

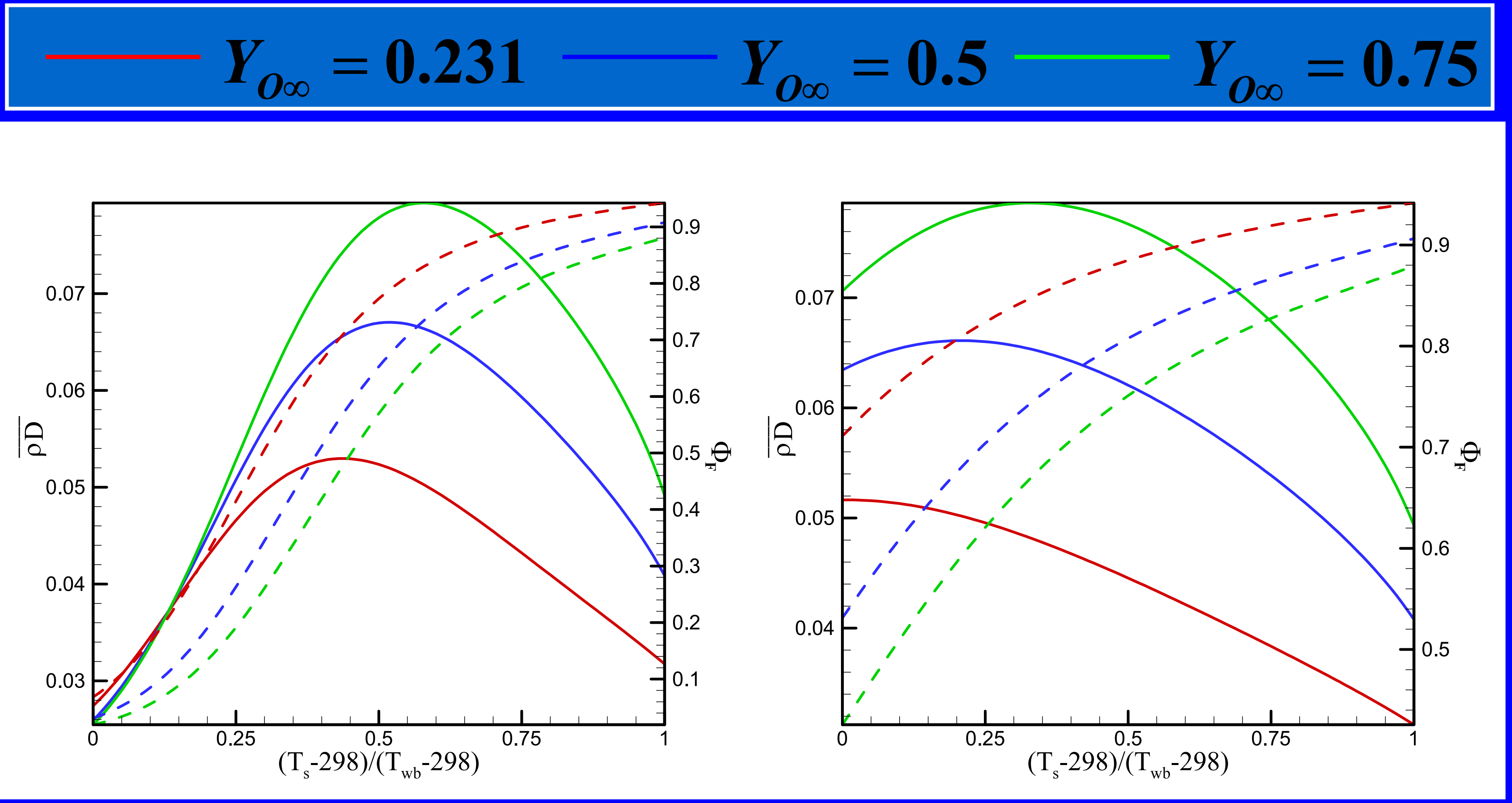
$$T_\infty = 298K,\ Y_{O\infty} = 0.231$$



Decane; T_{WB}=429.73K

Heptane; T_{WB}=359.36K

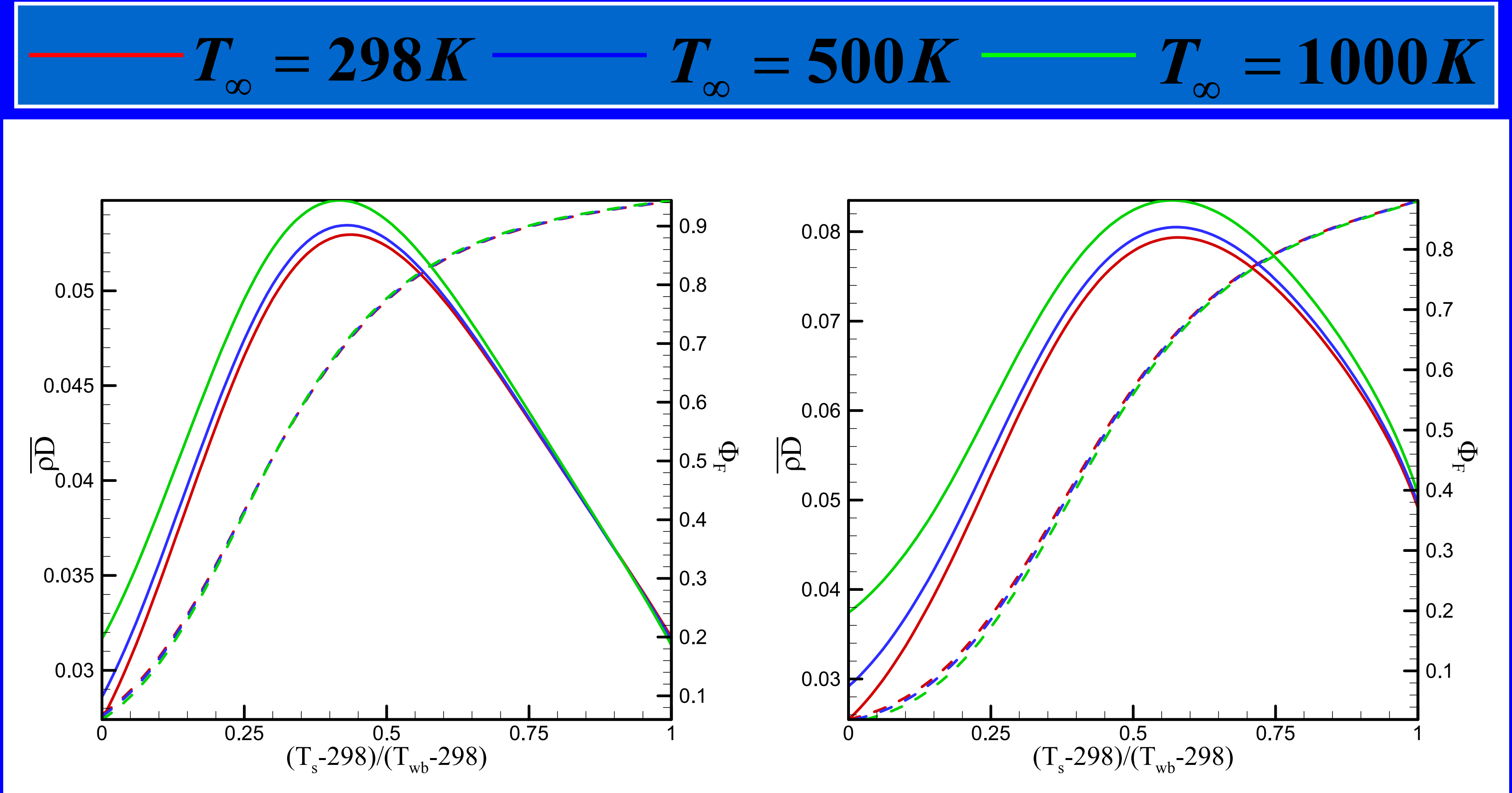
Effect of $Y_{O\infty}$ on $\overline{\rho D}$ (solid) and Φ_F (dashed)



Decane; $T_\infty = 298K$

Heptane; $T_\infty = 298K$

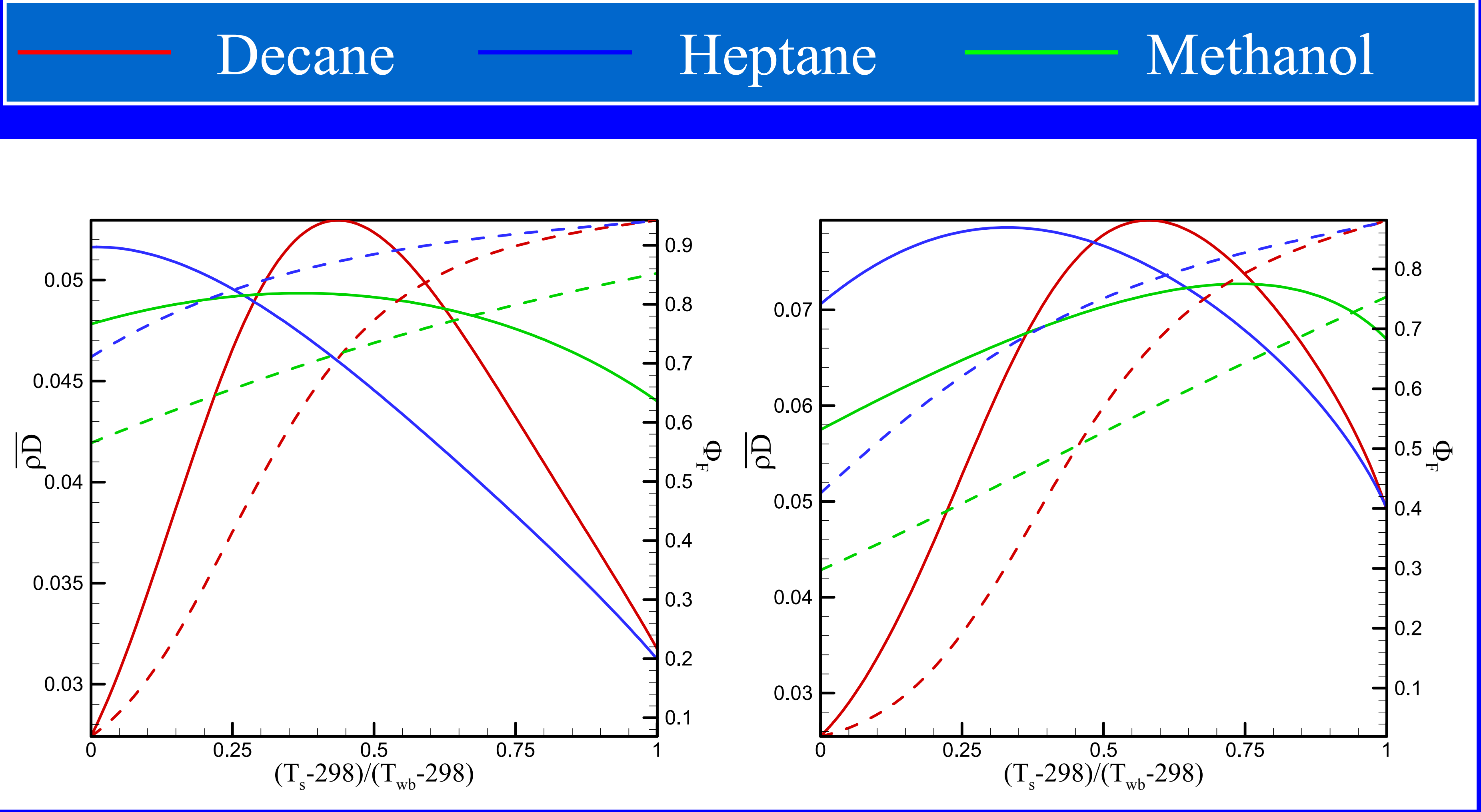
Effect of T_∞ on $\overline{\rho D}$ (solid) and Φ_F (dashed)



Decane; $Y_{O\infty} = 0.231$

Decane; $Y_{O\infty} = 0.75$

$\overline{\rho D}$ (solid) and Φ_F (dashed) for different fuels

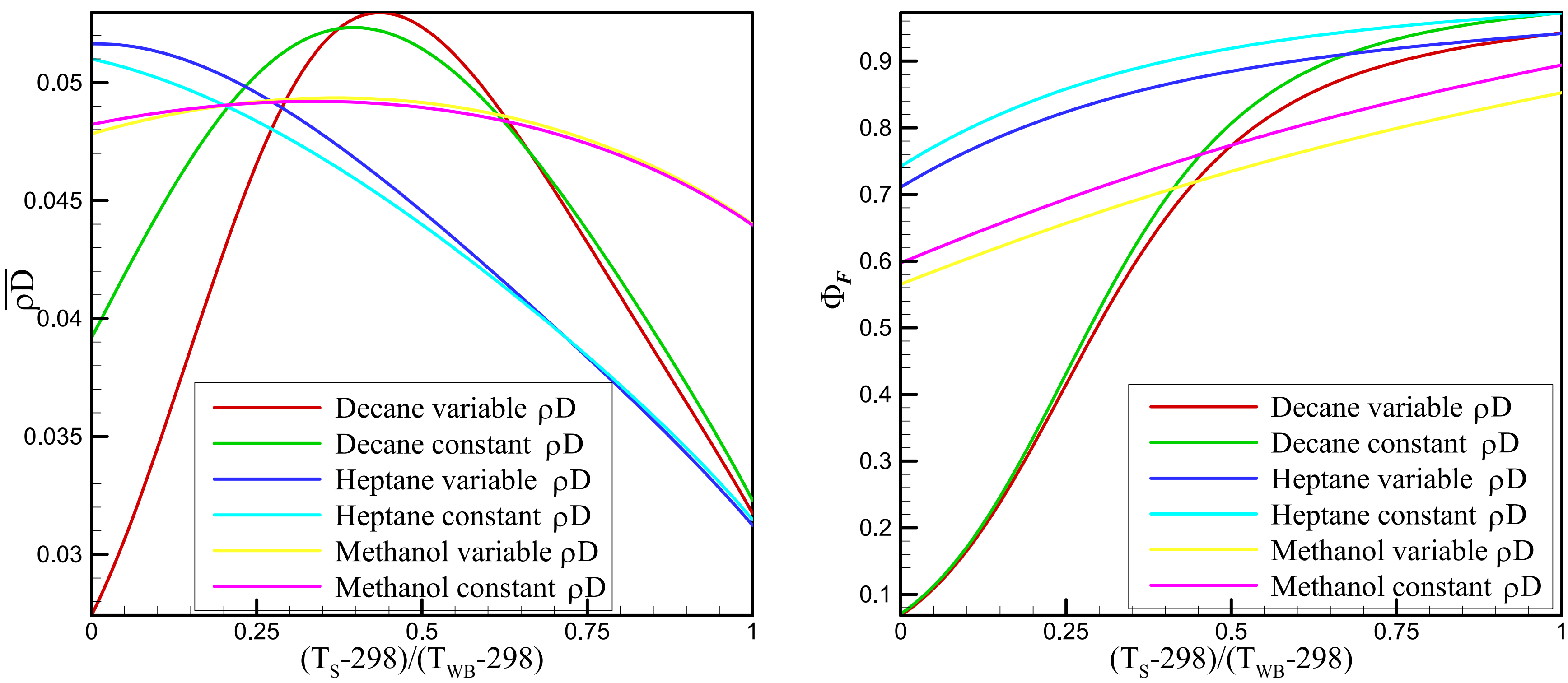


$T_\infty = 298K;\ Y_{O\infty} = 0.231$

$T_\infty = 298K;\ Y_{O\infty} = 0.75$

$\overline{\rho D}$ and Φ_F with constant and variable ρD

$T_\infty = 298K; Y_{O\infty} = 0.231$



Decane; T_{WB}=429.73K Heptane; T_{WB}=359.36K Methanol; T_{WB}=327.44K

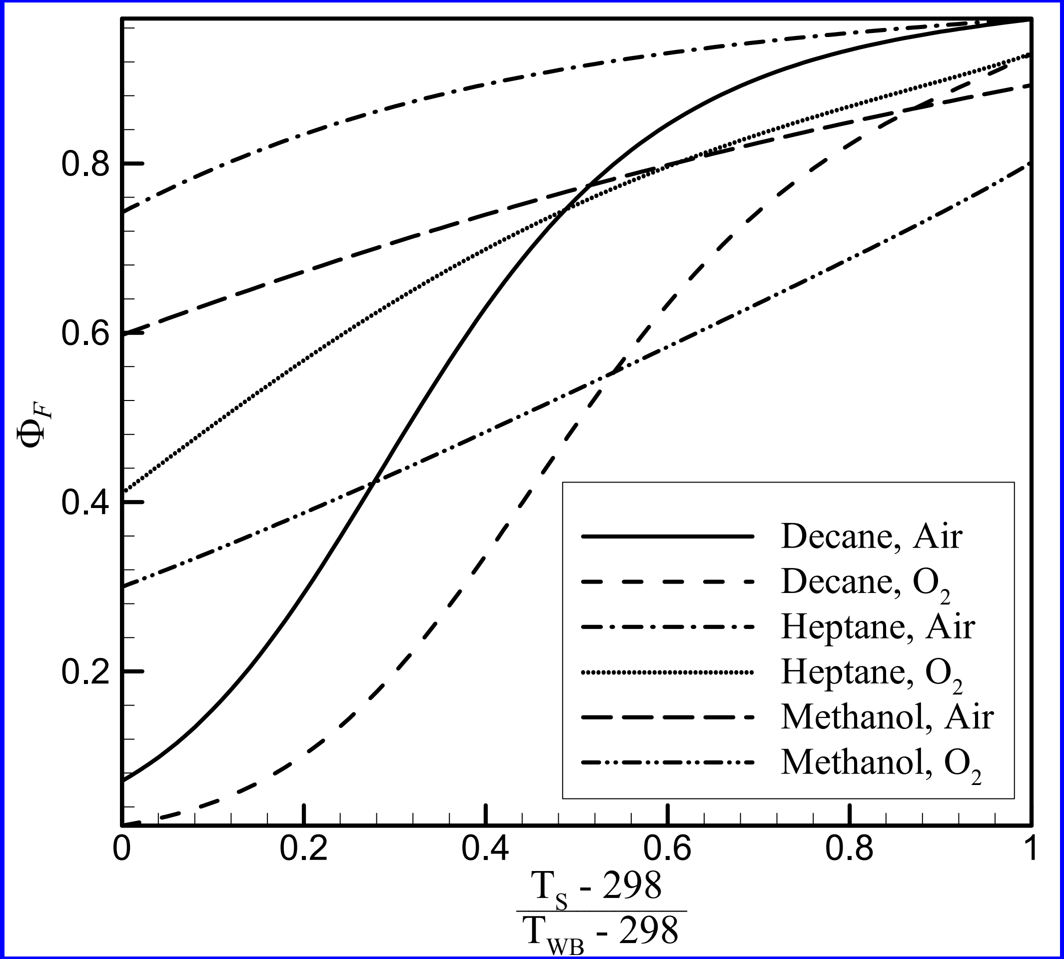
Factors Affecting Flame Position

- Droplet temperatures
- Ambient oxidizer mass fraction
- Type of fuel
- Ambient pressure
- Ambient temperature

Flames become approximately spherical at large distances from the droplet array

$$\Phi^*(r) = 1 - \frac{aN\eta_A}{r}$$

Flame locations in air and oxygen vs. droplet surface temperature

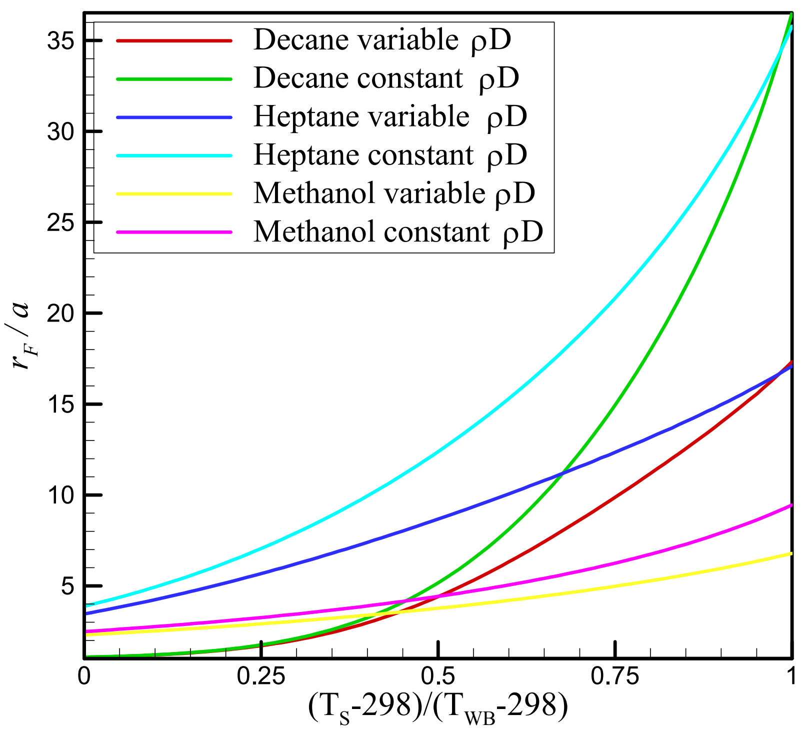


Droplets at wet-bulb temperatures had a spherical flame for all droplet arrays studied

Isolated droplet flame standoff comparison

$$T_\infty = 298K$$

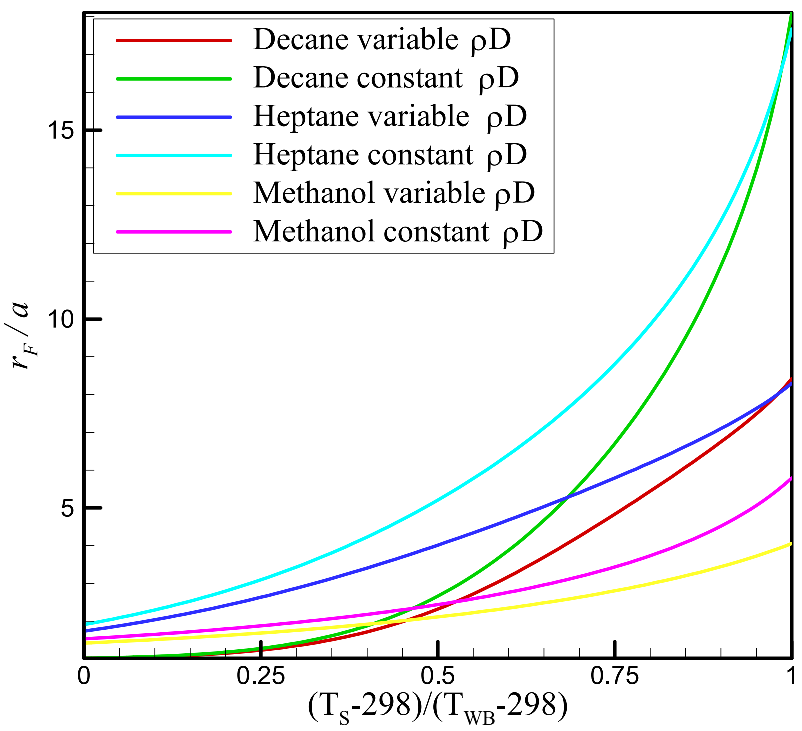
$$Y_{O\infty} = 0.231$$



Decane; T_{WB}=429.73K
Heptane; T_{WB}=359.36K
Methanol; T_{WB}=327.44K

$$T_\infty = 298K$$

$$Y_{O\infty} = 0.75$$



Decane; T_{WB}=440.62K
Heptane; T_{WB}=366.93K
Methanol; T_{WB}=332.97K

Comments

- **The scalar variables are a function of the mass-flux potential Φ without implicit dependence on position or time, universally valid for any configuration.**
- **The normalized Φ satisfies Laplace's equation without dependence on scalar constraints.**
- **The potential value of the thin flame and the flame temperature are independent of configuration.**
- **Variable Properties have important consequences.**
- **As we shall see, burning rate and vaporization rate do depend on configuration.**

The gas-phase for any geometry is governed by

$$\nabla^2\Phi = 0$$

with b. c.'s $\Phi = 0$ **on the liquid surfaces**

$\Phi = 1$ **far from the droplets**

Φ (x, y, z, t) does not depend on properties, scalar solutions, or boundary values.

Normalized droplet and array vaporization rates are

$$\eta_j = \frac{\dot{m}_j}{\dot{m}_{iso}} = \frac{1}{4\pi a_j}\iint \nabla\Phi \cdot d\vec{A}_j$$

$$\eta_A = \sum_{j=1}^{N} \dot{m}_j \Big/ N\dot{m}_{iso} = \frac{1}{4\pi a_j N}\sum_{j=1}^{N}\iint \nabla\Phi \cdot d\vec{A}_j = \frac{1}{N}\sum_{j=1}^{N}\eta_j$$

Vaporization Rates

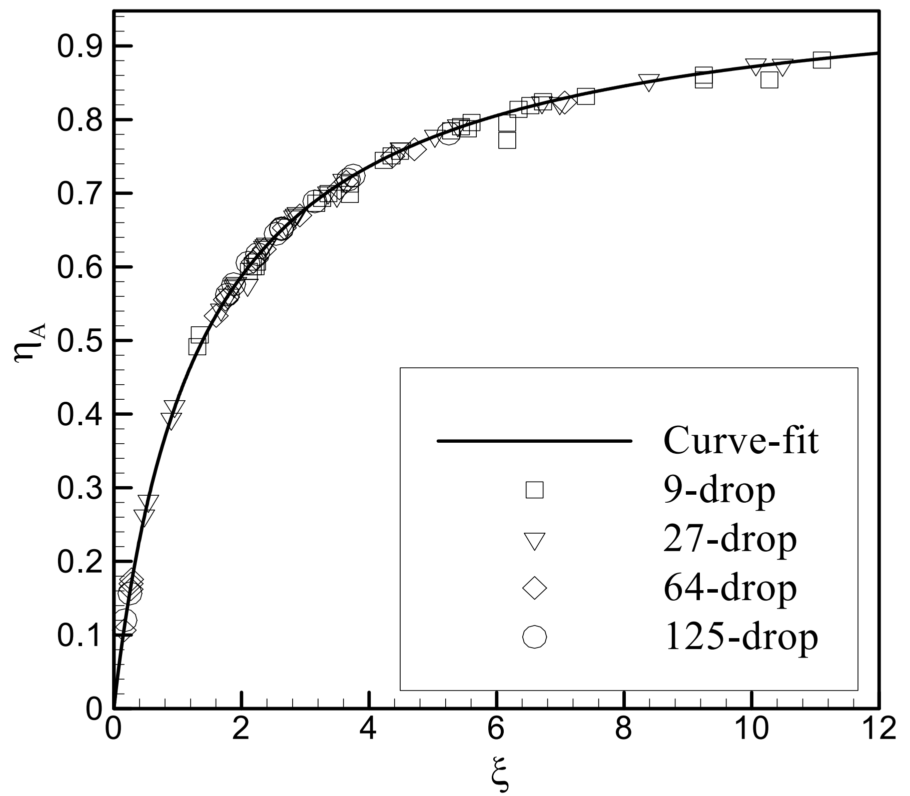
The similarity parameter ξ can be expressed in terms of the liquid / array volume ratio V_l / V_A and N

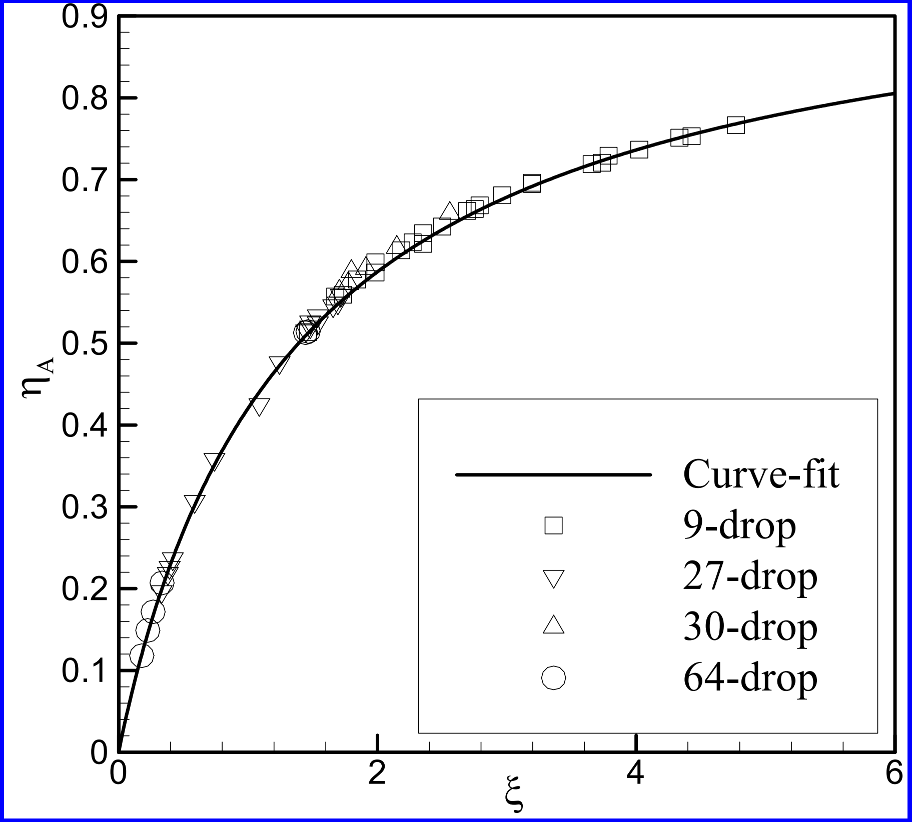
$$\xi = \frac{\left(\dfrac{4\pi V_A}{3V_l}\right)^{1/3}}{N^{0.72}}$$

Vaporization rate can be obtained from the correlation for η_A

$$\frac{\dot{m}_A}{\rho D \ln(1+B) V_A^{1/3}} = (4\pi N)^{2/3} \left(\frac{3V_l}{V_A}\right)^{1/3} \left\{1 - \frac{1}{1+0.7257\xi^{0.9717}}\right\}$$

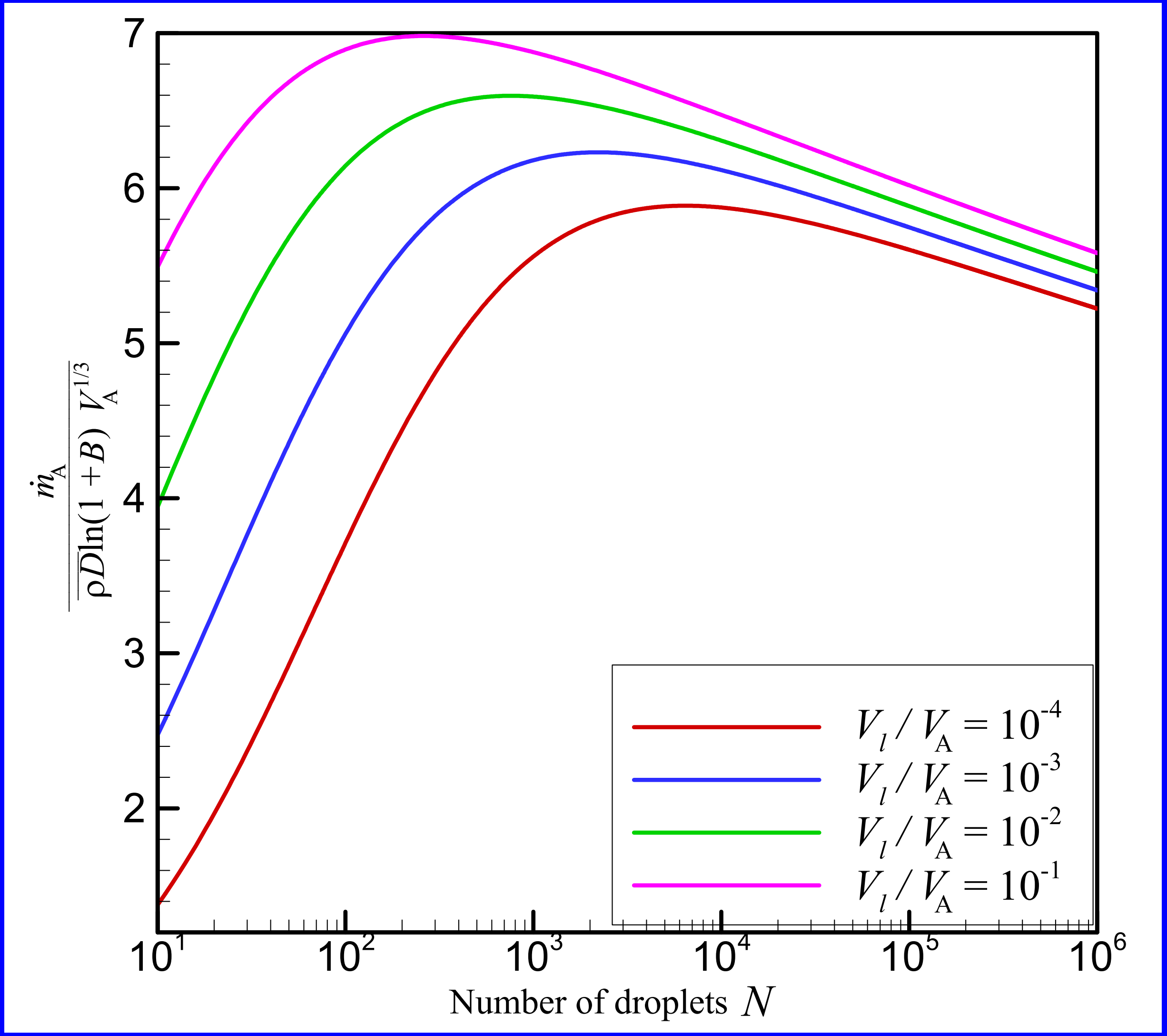
η_A vs. ξ for arrays with variable radii





η_A vs. ξ for arrays with non-uniform spacing

Dimensional normalized vaporization rate versus the number of droplets



Quasi-steady combustion at wet-bulb temperature

The vaporization rate of the j^{th} droplet at wet-bulb temperature is given by

$$\dot{m}_j = 4\pi a_j \overline{\rho D} \ln(1 + B_{WB})\eta_j = -\rho_l \frac{d}{dt}\left(\frac{4\pi a_j^3}{3}\right)$$

Using the dimensionless variables

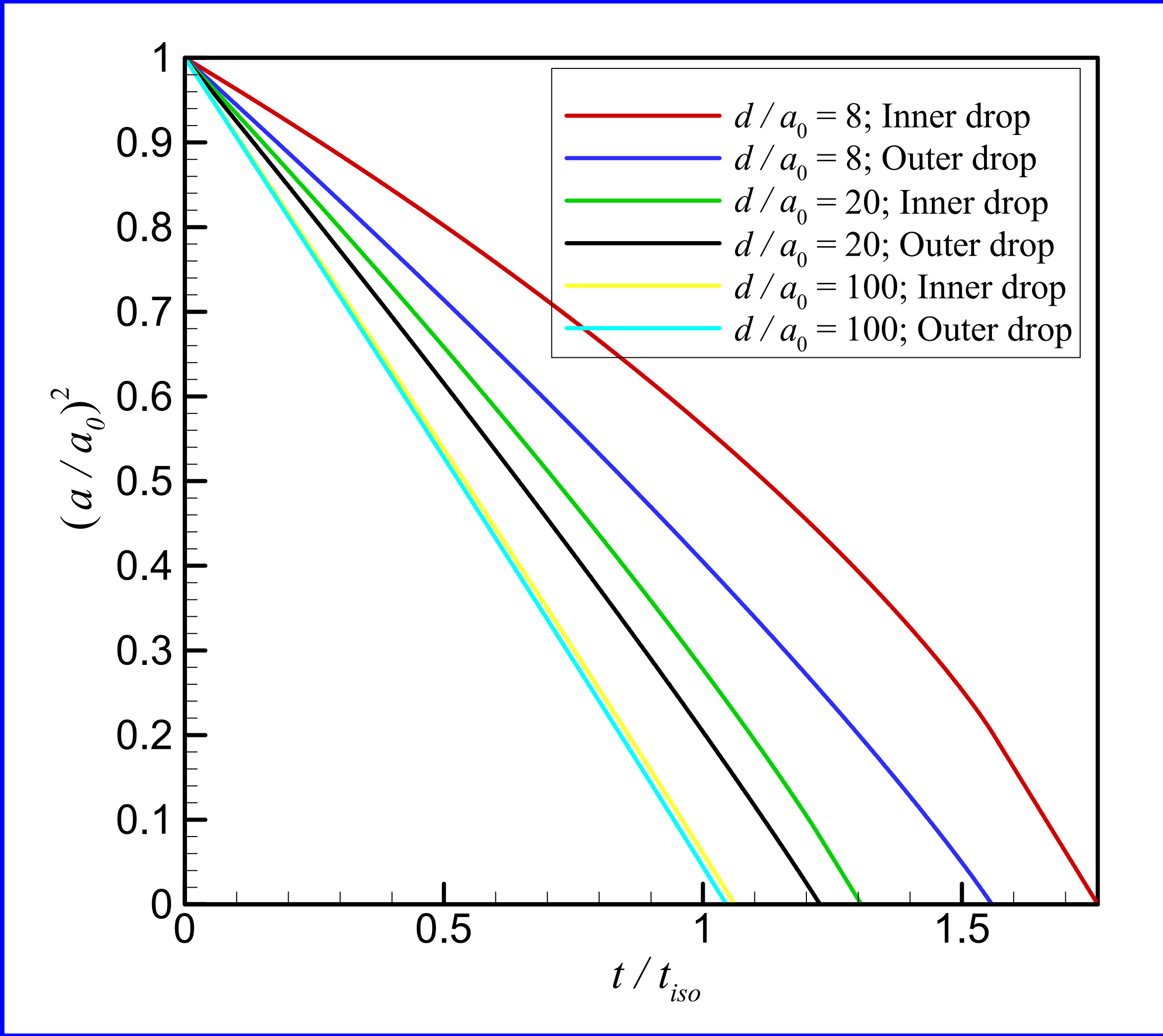
$$\hat{t} = \frac{t}{t_{iso}} \qquad \hat{a} = \frac{a}{a_0} \qquad t_{iso} = \frac{\rho_l a_0^2}{2\overline{\rho D}\ln(1 + B_{WB})}$$

The normalized droplet radius is governed by

$$\frac{d\hat{a}_j^2}{d\hat{t}} = -\eta_j$$

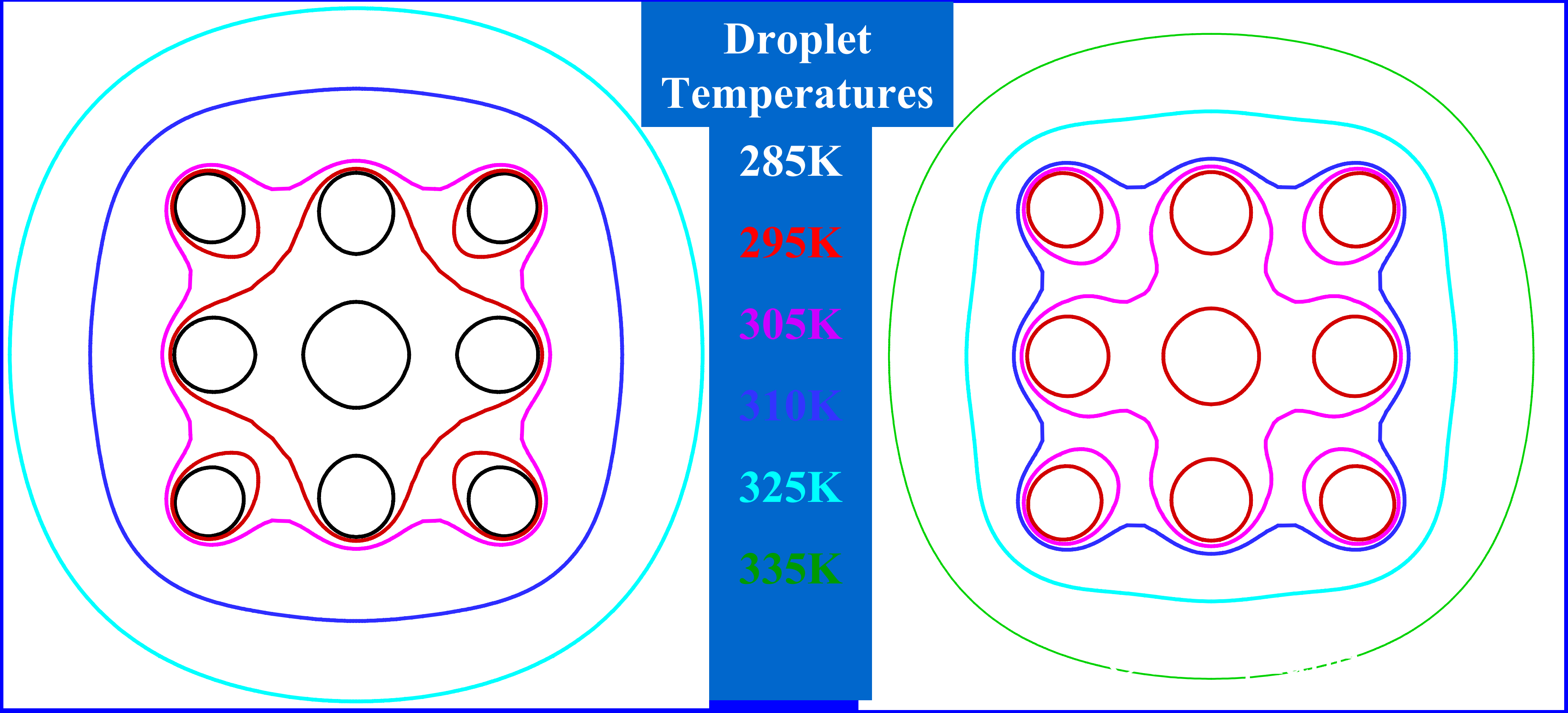
Quasi-steady combustion at wet-bulb temperature

- **Significant deviation from d^2 behavior due to interactions**
- **Droplet vaporization rates depend strongly on inter-droplet spacing**
- **Individual droplet burning rates integrated using a 4th order RK scheme**
- **Peripheral droplets burn faster than inner droplets**
- **Fuel type and boundary conditions for a fixed initial geometry affect results only through t_{iso}**



Square of normalized radius vs. time for 9-drop cubic arrays with different initial spacing

Flame locations in the central and outer planes of a 27-drop cubic array with d/a = 5 for decane/air at various droplet temperatures($T_{wet-bulb} = 430.8K$)



Droplet Temperatures

285K

295K

305K

310K

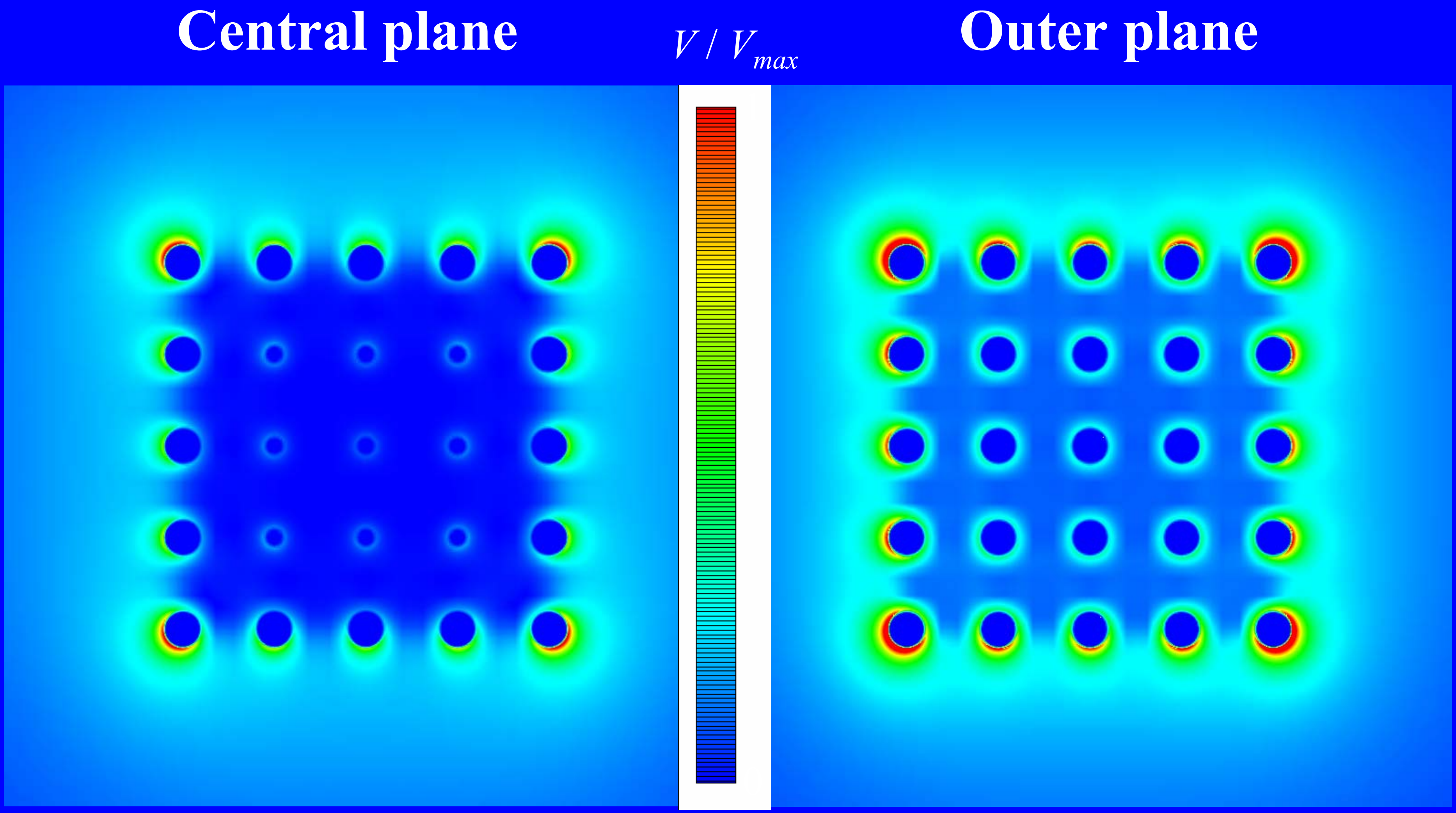
325K

335K

- 285K: 27 individual-droplet flames
- 295K: 7 droplets burn as a group plus 20 individual-droplet flames
- 305K: 19 droplets burn as a group plus 8 individual-droplet flames
- 310K: 27 droplets burn as a group

Stefan velocity in the central and outer planes of a 125-drop cubic array with $d / a_{outer} = 5$ and $a_{inner} / a_{outer} = 0.5$

27 inner, 98 outer droplets



Remarks

- Peripheral droplets can vaporize substantially faster than inner droplets
- Arrays of different size and spacing can be characterized by a similarity parameter
- An optimal number of droplets exists and depends only on the fuel / array volume ratio
- Droplet lifetimes are increased due to interactions
- Different modes of combustion exist depending on fuel, droplet temperatures, and ambience
- Individual droplet burning was not observed for droplets at wet-bulb temperatures
- A spherically symmetric solution exists for large distances from the droplet array

Unsteady combustion with liquid heating

- Liquid surface temperature is spatially uniform and equal for all droplets which implies that the effects of droplet interactions are the same for all droplets in the array.

- Liquid temperature distribution can be approximated by one of the following models:
    - Conduction-Limit: Liquid temperature varies radially and temporally within the droplet
    - Infinite Liquid Conductivity: Liquid temperature varies temporally but is spatially uniform

Unsteady combustion with liquid heating

The diffusion equation in the liquid

$$\frac{\partial T}{\partial t} = \frac{\alpha_l}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)$$

is normalized by

$$\hat{t} = \frac{t}{t_{iso}} \qquad \hat{r} = \frac{r}{a} \qquad \hat{a} = \frac{a}{a_0}$$

Liquid-phase temperatures are now governed by

$$\frac{\partial T}{\partial \hat{t}} = \frac{\alpha_l t_{iso}}{(a_0\hat{a})^2}\frac{\partial^2 T}{\partial \hat{r}^2} + \left(\frac{2\alpha_l t_{iso}}{(a_0\hat{a})^2\hat{r}} + \frac{\hat{r}}{2\hat{a}^2}\frac{d\hat{a}^2}{d\hat{t}}\right)\frac{\partial T}{\partial \hat{r}}$$

Droplet radii are obtained by solving

$$\frac{d\hat{a}^2}{d\hat{t}} = -\frac{\ln(1+B)}{\ln(1+B_{WB})}\eta_A$$

Unsteady combustion with liquid heating

The droplet surface boundary condition is computed from the energy balance at interface plus heat conduction from gas to surface.

No temperature gradient at the droplet center

Then, the average energy flux into the liquid is given by

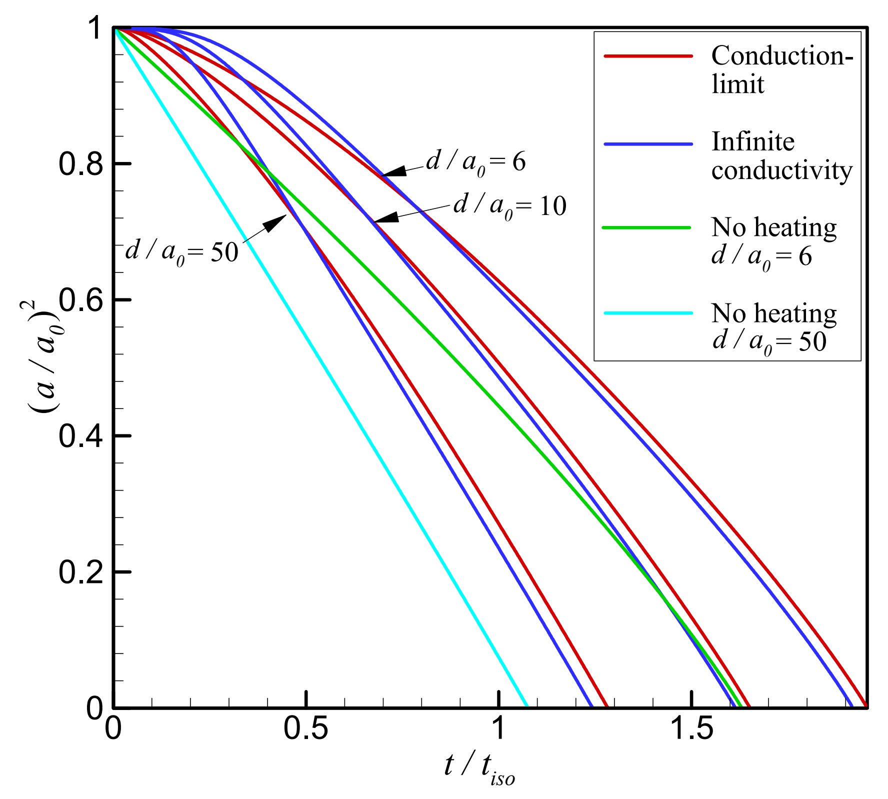
$$\bar{\dot{q}}_l = \frac{1}{4\pi a^2}\iint \dot{q}_l \cdot d\vec{A} = \frac{\lambda_l}{a}\frac{\partial T}{\partial \hat{r}}\bigg|_{\hat{r}=1}$$

$$\bar{\dot{q}}_l = \frac{\overline{\rho D}}{a}\ln(1+B)\left(\frac{h_\infty - h_S + \nu Q Y_{O\infty}}{B} - L\right)\eta_A$$

Correlations for $\overline{\rho D}(T_S)$ and $\Phi_F(T_S)$ are used throughout the time integration.

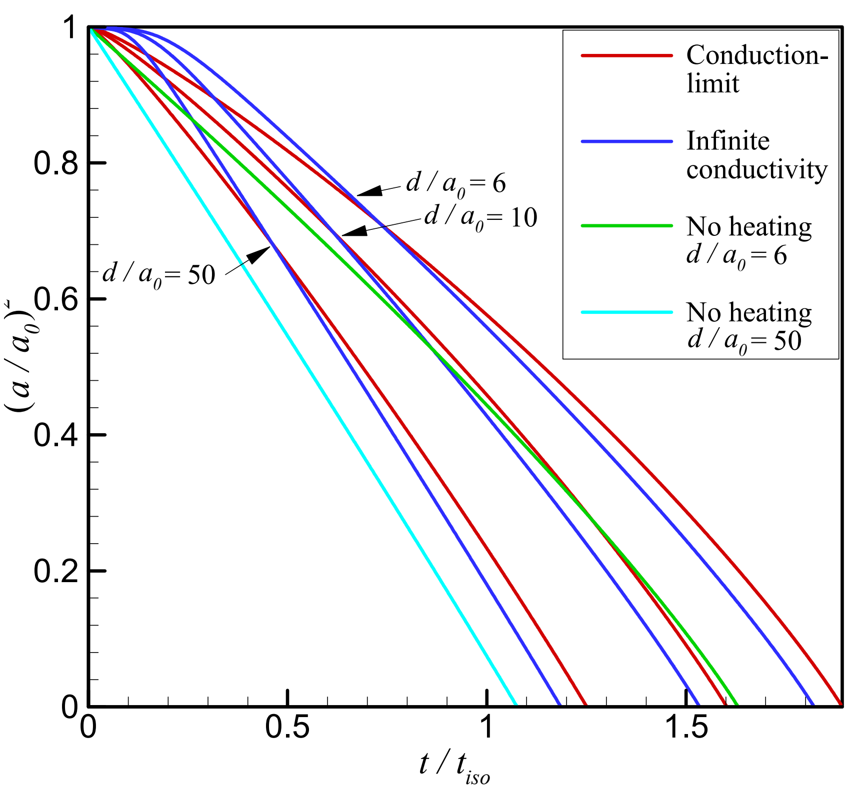
Square of normalized radius vs. time for cubic arrays of 8 decane drops with different initial spacing

$$T_\infty = 298K;\ Y_{O\infty} = 0.231$$



$$\frac{t_{iso}}{a_0^2} = 5.058716\left(\frac{\text{sec}}{\text{mm}^2}\right)$$

$$T_\infty = 298K;\ Y_{O\infty} = 0.75$$

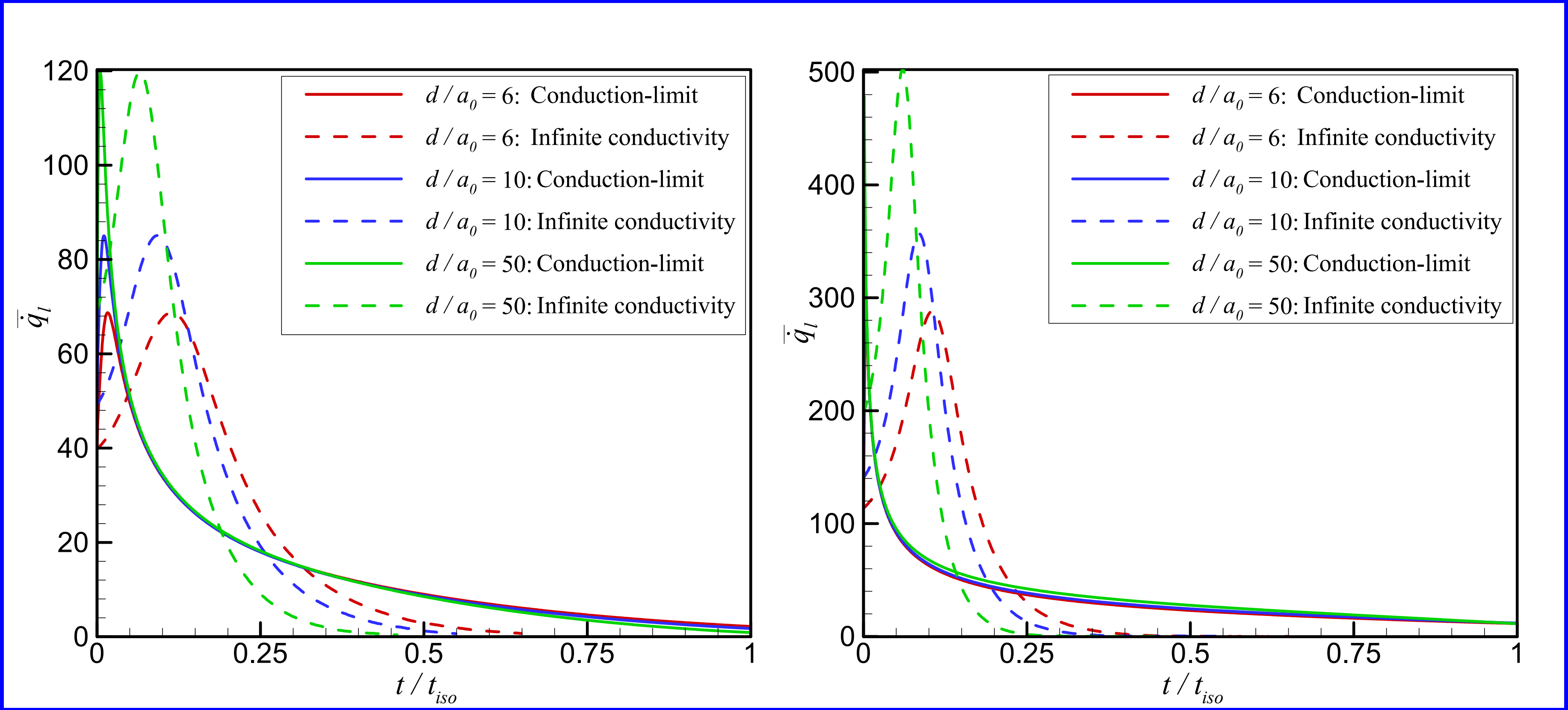


$$\frac{t_{iso}}{a_0^2} = 2.121125\left(\frac{\text{sec}}{\text{mm}^2}\right)$$

Heat flux into the liquid (kW/m^2) vs. time for cubic arrays of 8 decane drops with different initial spacing

$T_\infty = 298K;\ Y_{O\infty} = 0.231$

$T_\infty = 298K;\ Y_{O\infty} = 0.75$



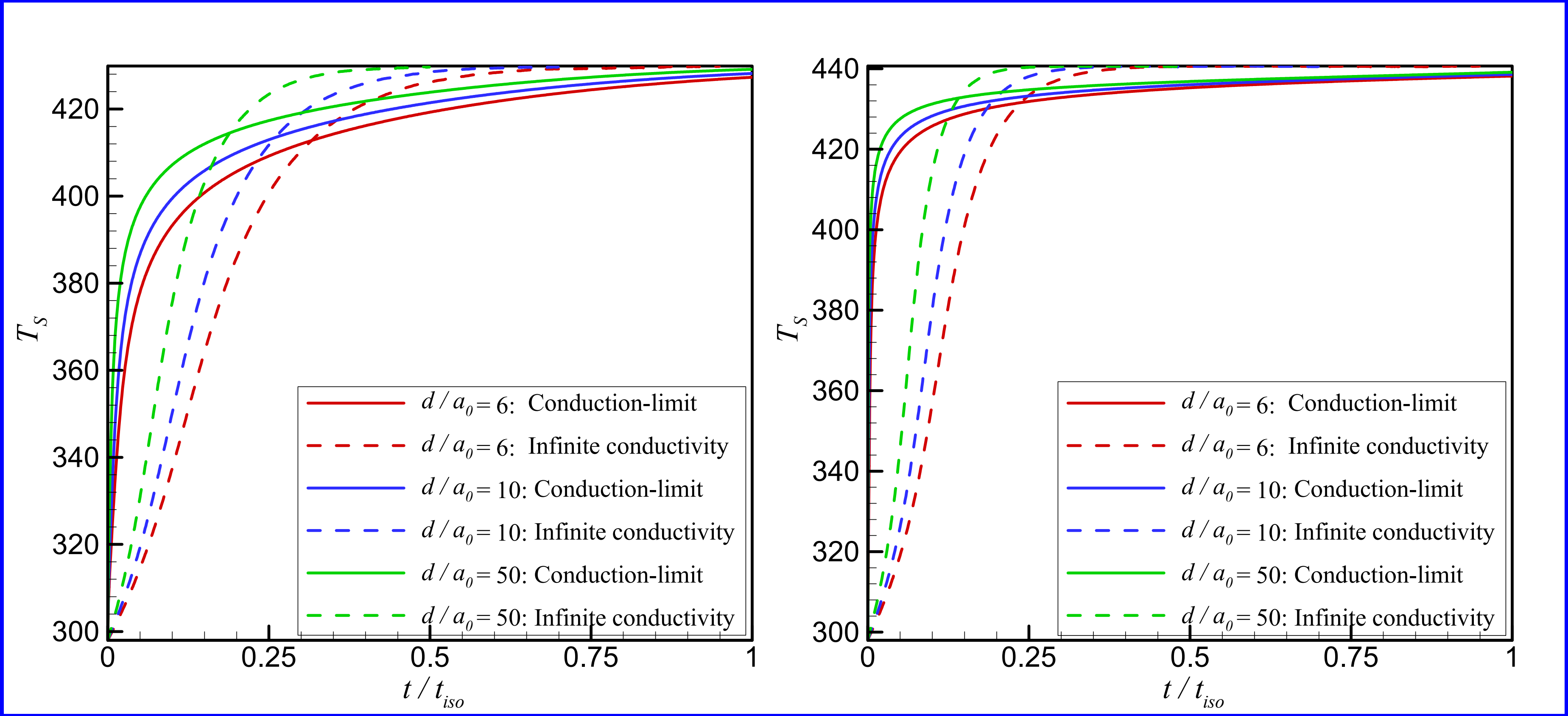
$$\frac{t_{iso}}{a_0^2} = 5.058716\left(\frac{\text{sec}}{\text{mm}^2}\right)$$

$$\frac{t_{iso}}{a_0^2} = 2.121125\left(\frac{\text{sec}}{\text{mm}^2}\right)$$

Droplet surface temperature vs. normalized time for cubic arrays of 8 decane drops with different initial spacing

$T_\infty = 298K; Y_{O\infty} = 0.231$

$T_\infty = 298K; \ Y_{O\infty} = 0.75$

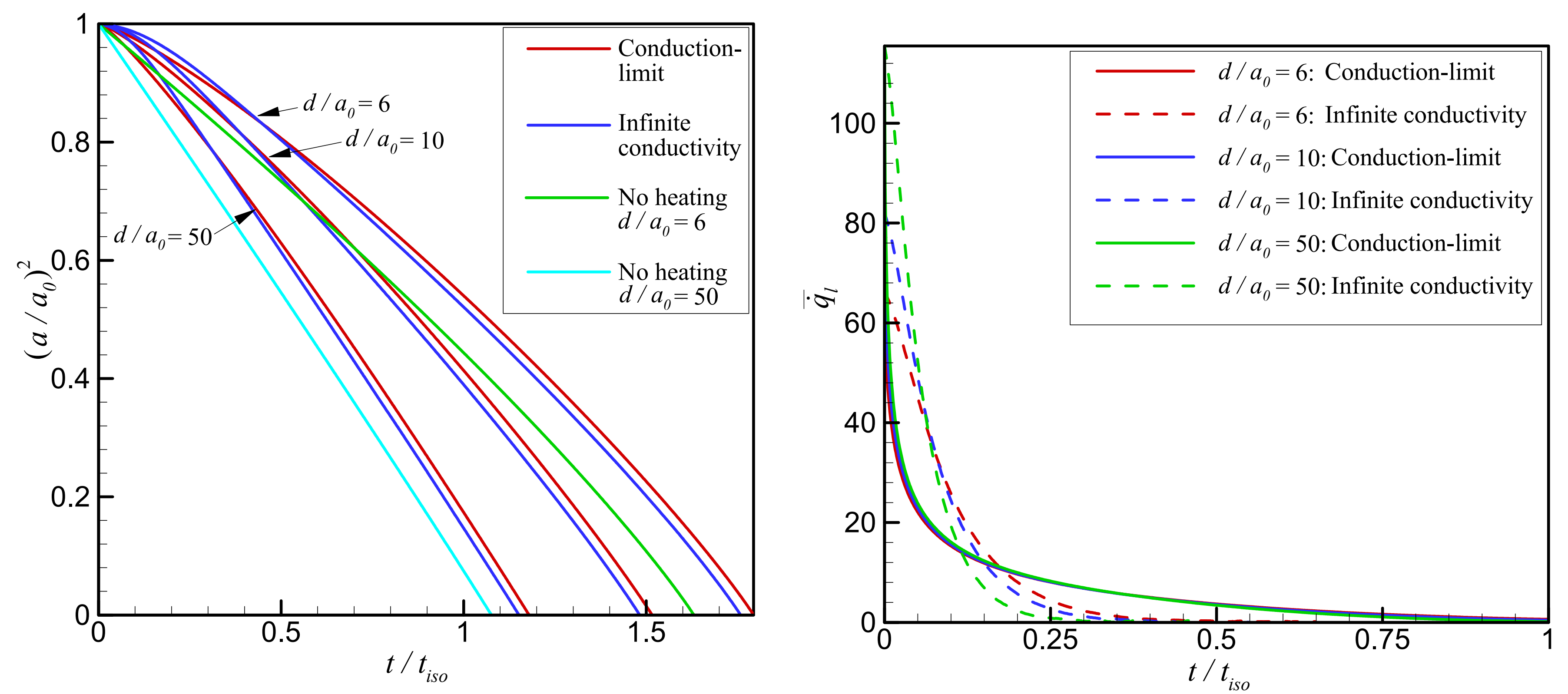


$$\frac{t_{iso}}{a_0^2} = 5.058716\left(\frac{\text{sec}}{\text{mm}^2}\right) \quad T_{WB} = 429.73$$

$$\frac{t_{iso}}{a_0^2} = 2.121125\left(\frac{\text{sec}}{\text{mm}^2}\right) \quad T_{WB} = 440.62$$

Square of normalized radius and heat flux (kW/m^2) vs. time for cubic arrays of 8 heptane drops with different initial spacing

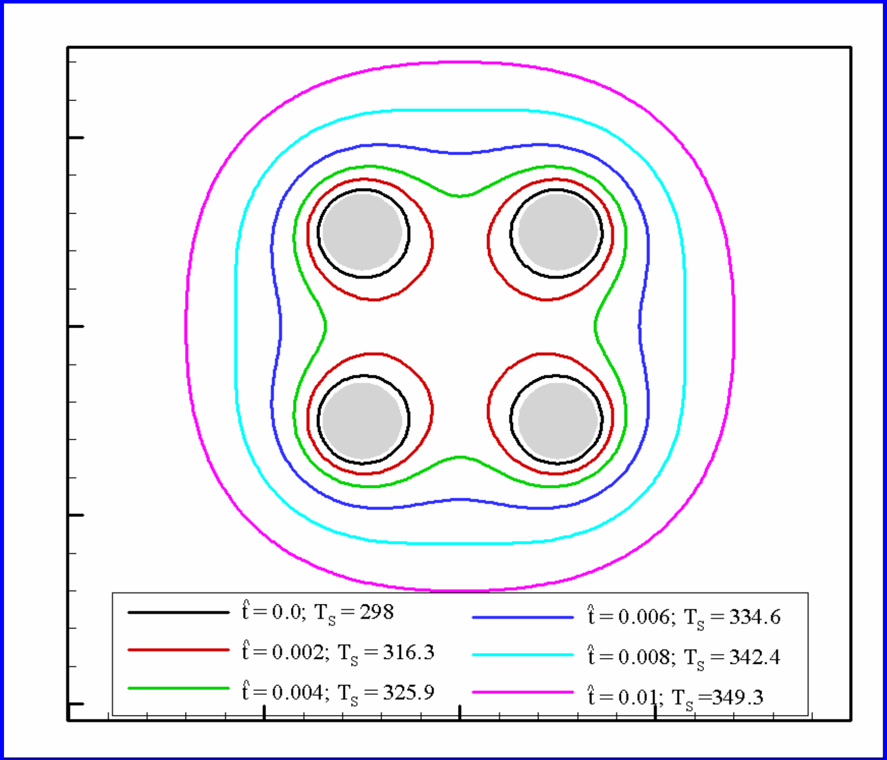
$$T_\infty = 298K;\ Y_{O\infty} = 0.231$$



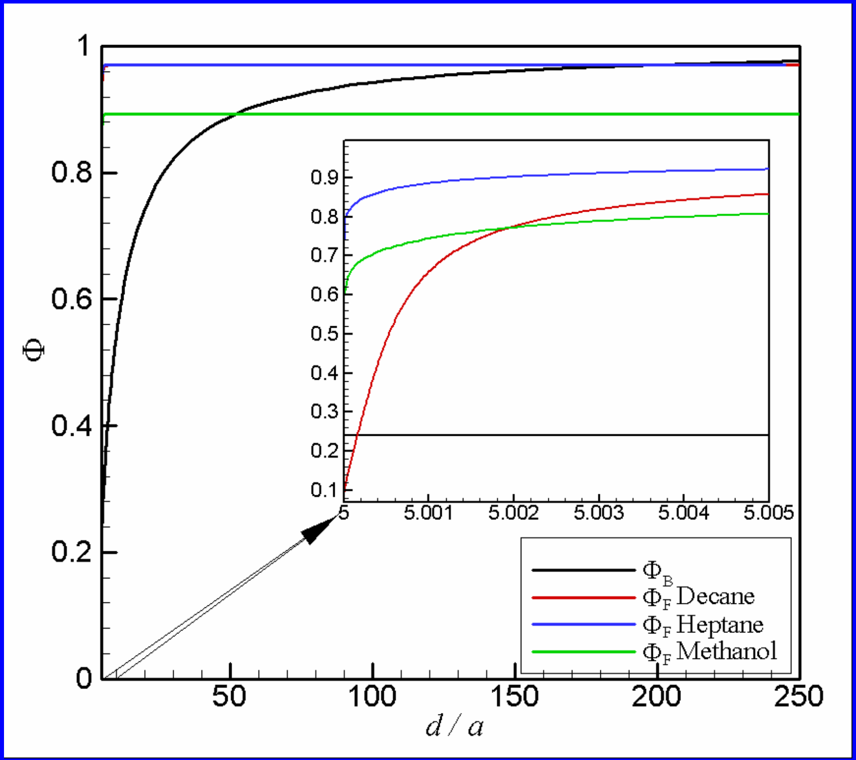
Flame Shape: Group vs. Individual Flame

4-drop array ; $d/a_0 = 5$; conduction-limit heating model ; $T_0 = 298$ K

Decane Fuel – T_s and Φ_F increase with time



Φ_F for various fuels and bifurctation value Φ_B

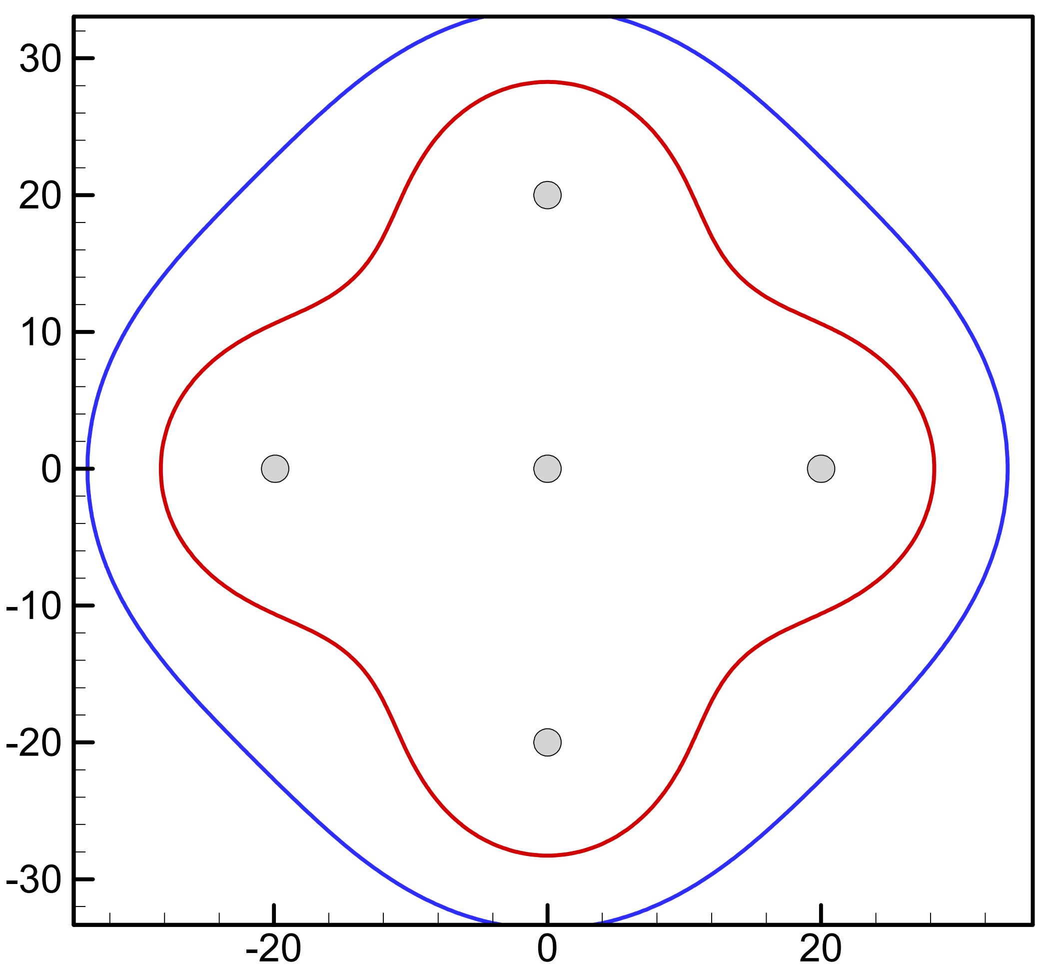


Conclusions

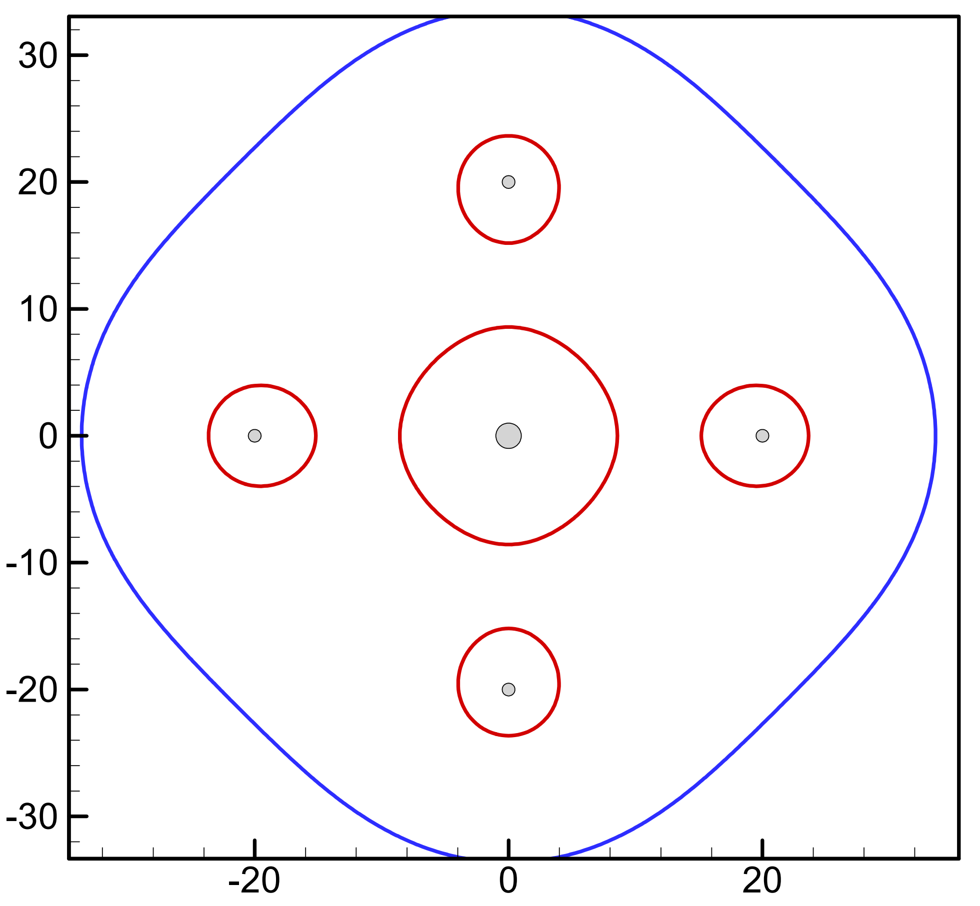
- The scalar variables are a function of the mass-flux potential Φ without implicit dependence on position or time, universally valid for any configuration.
- The normalized Φ satisfies Laplace's equation without dependence on scalar constraints.
- Vaporization rates for arrays of different sizes and spacings depend only on a similarity parameter.
- Droplet lifetimes are increased due to interactions and / or due to transient droplet heating.
- Different combustion modes (group or individual-droplet) exist depending on fuel, configuration, droplet temperatures, and ambient conditions.
- Combustion modes can change during the array lifetime due to decreasing droplet size and / or droplet heating.

Thank you for your attention.

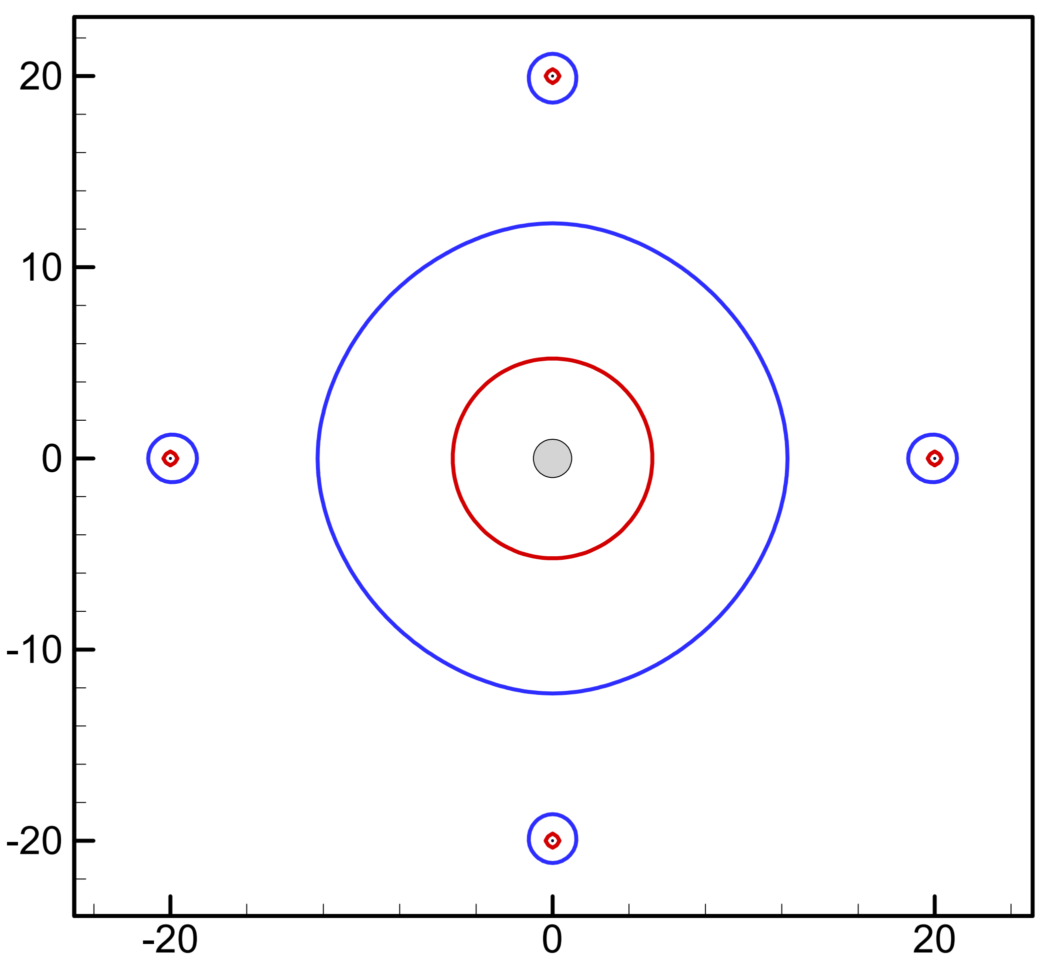
Flame locations for methanol (red) and decane (blue) droplets in a 5-drop array with a_i / a_o = 1 and spacing of d / a_i = 20.



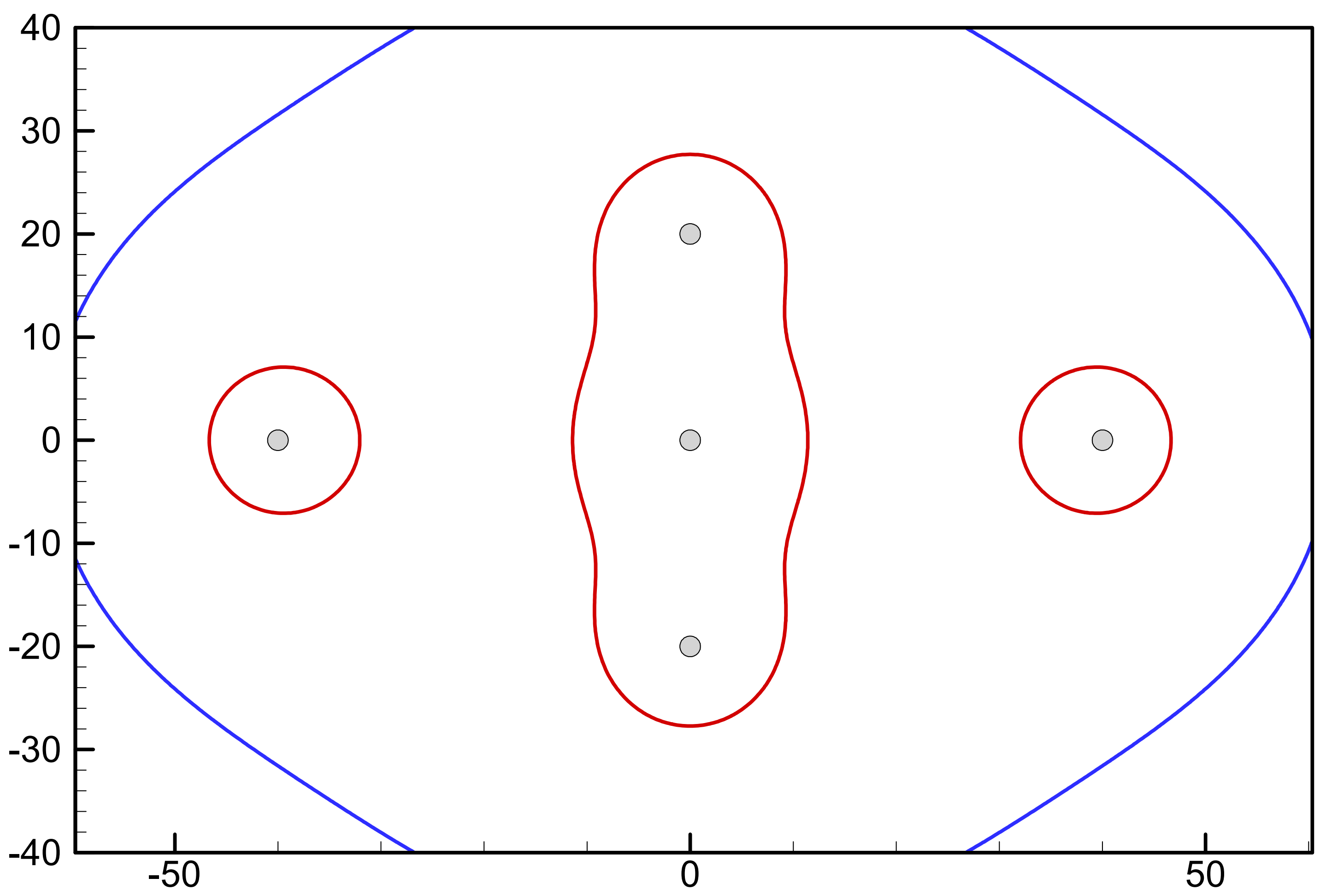
Flame locations for methanol (red) and decane (blue) droplets in a 5-drop array with a_i / a_o = 2 and spacing of d / a_i = 20.



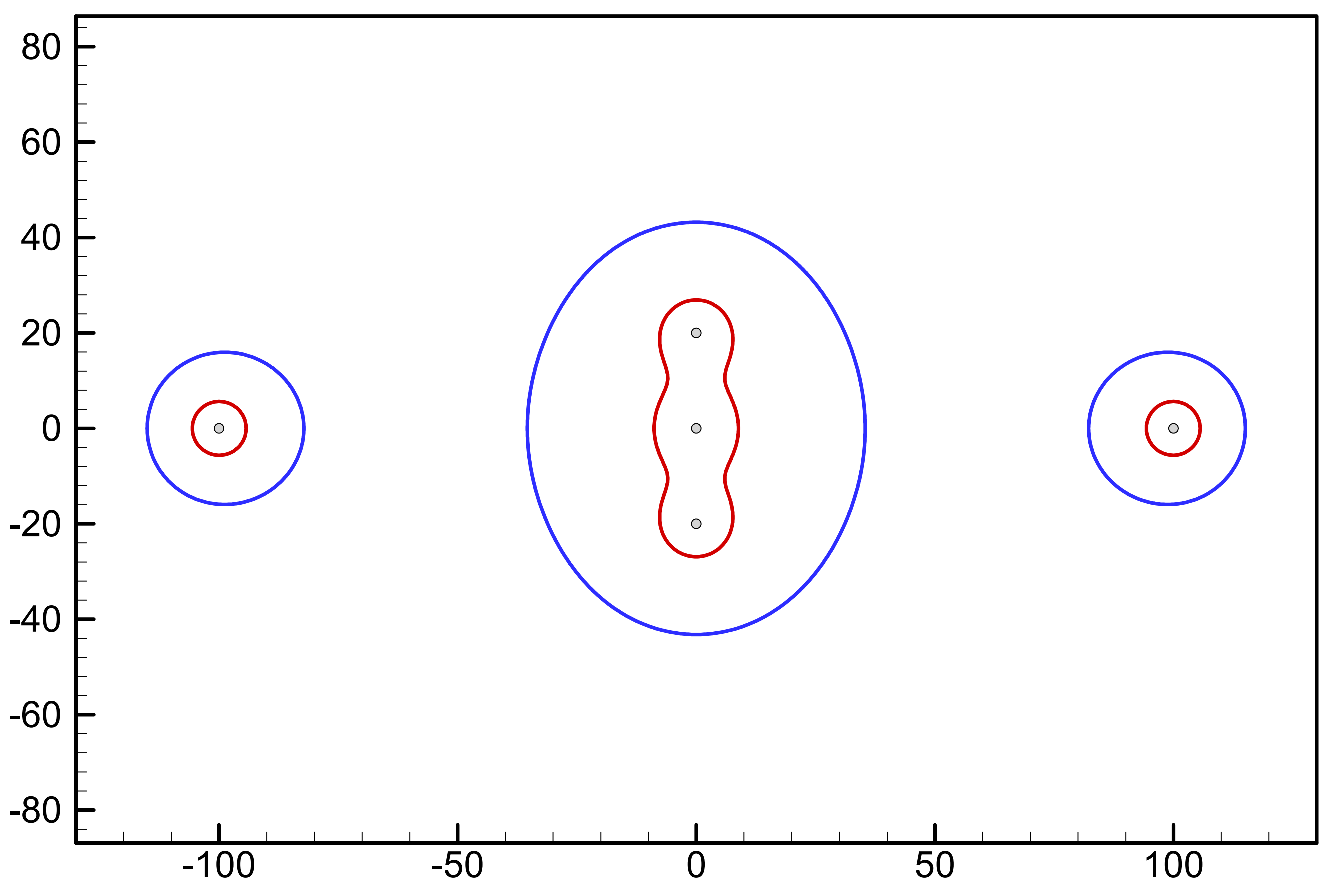
Flame locations for methanol (red) and decane (blue) droplets in a 5-drop array with a_i / a_o = 20 and spacing of d / a_i = 20.



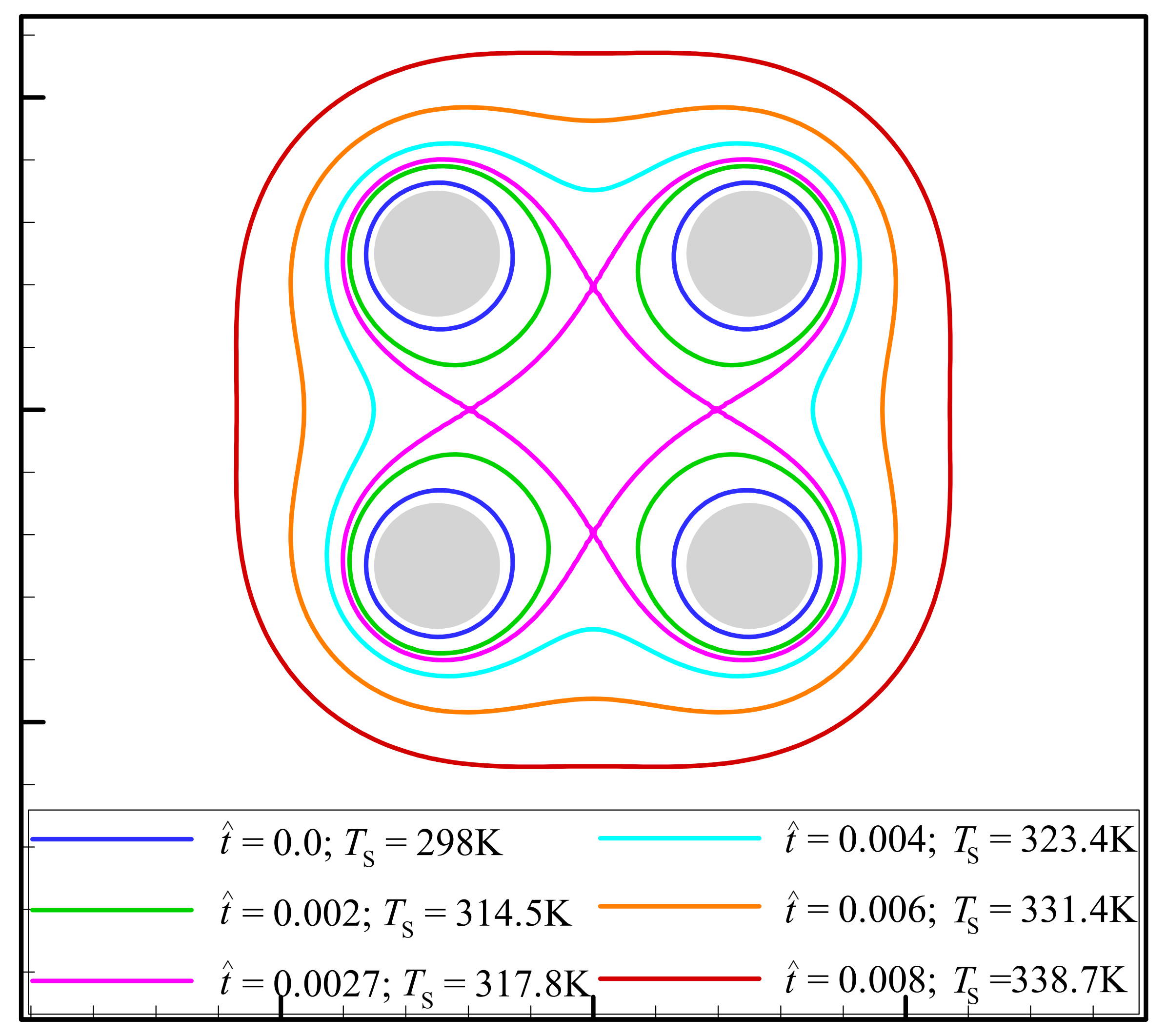
Flame locations for methanol (red) and decane (blue) droplets in a 5-drop array with $d_{min} = 20$ and $d_{max} / d_{min} = 2$.



Flame locations for methanol (red) and decane (blue) droplets in a 5-drop array with $d_{\min} = 20$ and $d_{\max} / d_{\min} = 5$.

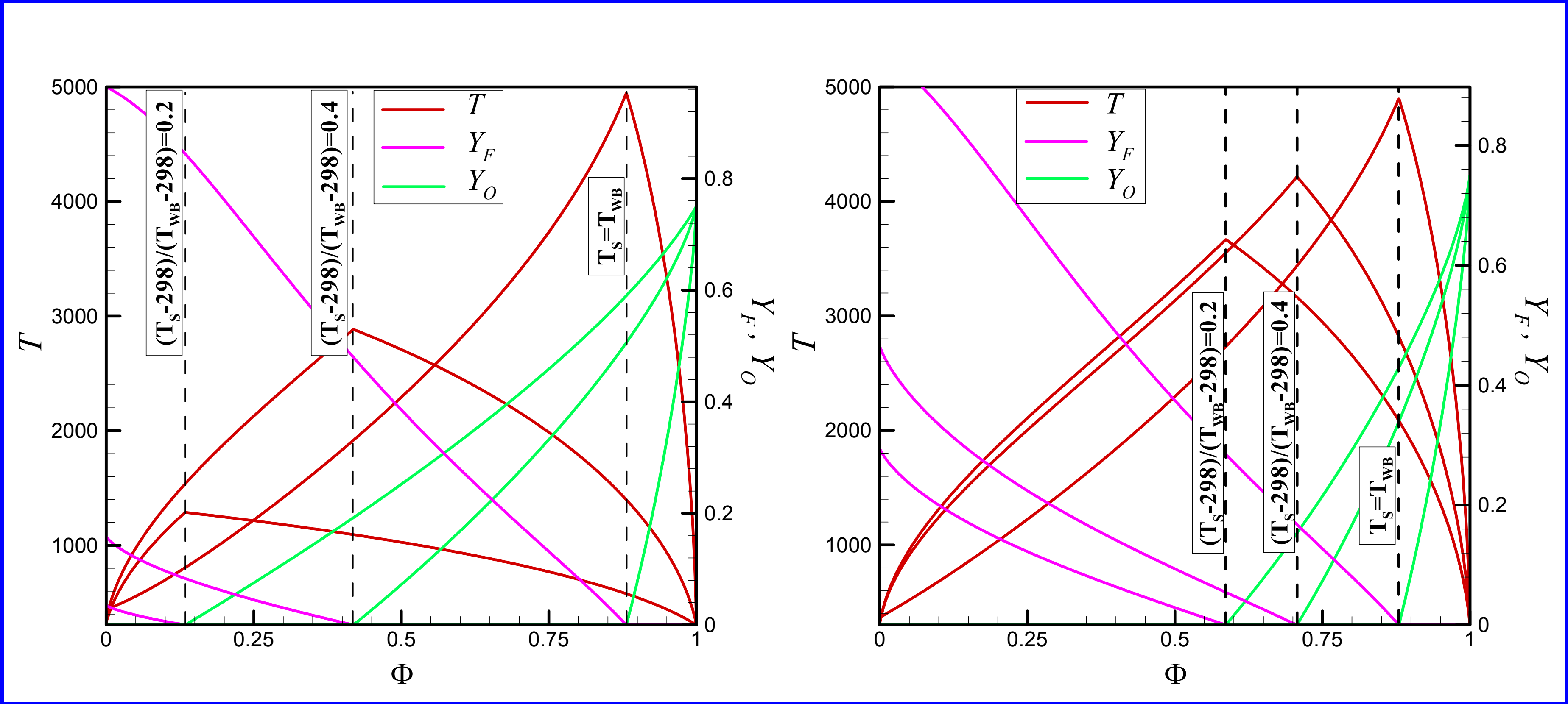


Flame contours for a 4-drop, decane array with $d/a_0 = 5$, T_0 = 298K, T_∞ = 298K, and $Y_{O\infty}$ = 0.231. Conduction-limit model.



Mixture temperature, fuel and oxidizer mass fractions for decane and heptane

$$T_\infty = 298K,\ \ Y_{O\infty} = 0.75$$



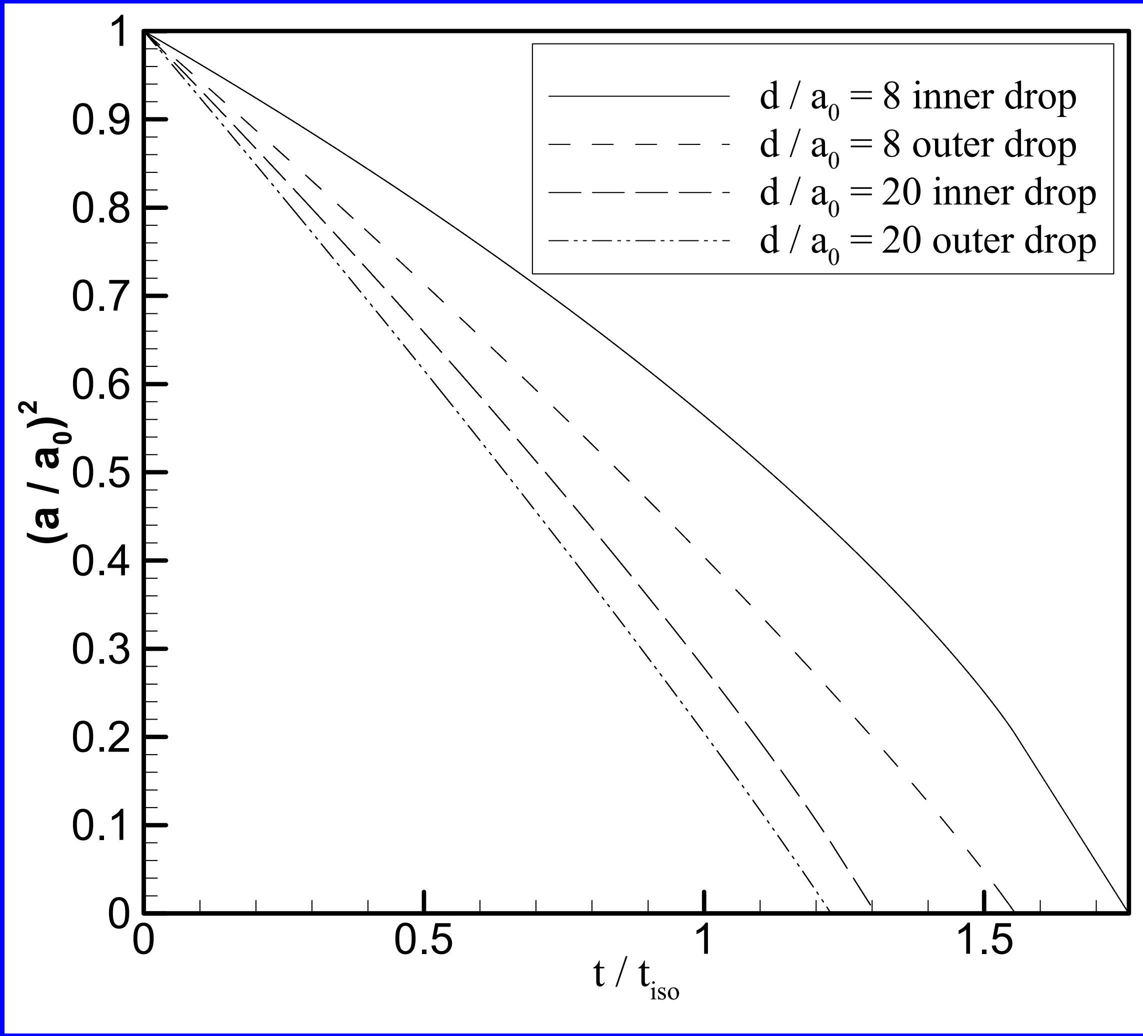
Decane; T_{WB}=440.62K

Heptane; T_{WB}=366.93K

Droplet Lifetime

- **Individual droplet burning rates integrated using a 4th order RK scheme**
- **Significant deviation from d^2 behavior due to interactions**
- **Liquid-phase heating will further increase droplet lifetimes**

Square of normalized droplet radius vs. dimensionless time for 9-drop, cubic arrays



Isolated Droplet Burning

Consider steady or unsteady spherically-symmetric, forced-convective, natural-convective, or mixed convective droplet vaporization and/or burning.

- ***Calculate temperature and vapor mass fraction at liquid-surface.***

$$\frac{h_\infty - h_s + \nu Q Y_{O_\infty}}{L_{eff}} = \frac{Y_{F_S} - Y_{F_\infty} + \nu Y_{O_\infty}}{1 - Y_{F_S}} \equiv B$$

- ***Calculate limiting diffusion-flame temperature.***

$$h(T_f) = h(T_\infty) + \nu\left(Q - \frac{L_{eff}}{1 - Y_{Fs}}\right) Y_{O\infty} + \frac{L_{eff}}{1 - Y_{Fs}} Y_{F\infty}$$

$$= h(T_s) + \frac{L_{eff}}{1 - Y_{Fs}(T_s)} Y_{Fs}(T_s)$$

- ***Above calculations are done simultaneously with use of phase-equilibrium law and liquid heat-transfer analysis.***

- ***Above calculations do not require prior determinations of the gas-phase velocity or scalar fields.***

Isolated Droplet Burning (continued)

- ***Results agree with textbook calculations for quasi-steady, spherically symmetric droplet burning.***
- ***Unsteady spherically symmetric droplet burning results agree with Waldman and Botros et al. analyses: e.g., constant temperature at moving flame.***
- ***Convective droplet results agree with published results of Sirignano and co-workers and of others.***
- ***Extension to multicomponent-fuel case also results in agreement with previously published results.***

Droplet Array Burning

- *Previous authors of two droplet vaporization, Twardus & Brzustowski and Umemura et al. did not report flame temperature.*

- *Present analysis predicts a flame temperature that is independent of droplet spacing or sizes and of whether droplets are equi-sized or not.*

- *Determination of flame shape however requires complete solution of scalar fields.*

- *Neglect of Stefan convection requires replacement of* $L_{eff} / (1-Y_{Fs}(T_s))$ *by* L_{eff} *in formulation of the super scalar* S.

Spray Analysis

Develop multicontinua formulation.

$$L(Y_i) = \rho w_i + \delta_i / \theta$$

$$L(h) = -\rho \dot{w}_F Q + \frac{1}{\theta}\frac{\partial(\theta p)}{\partial t} - \gamma / \theta$$

where

$$\gamma = \sum_k n^{(k)} \dot{m}^{(k)} \left[h - h_s^{(k)} + L_{eff}^{(k)} \right]$$

and

$$\delta_i = 0$$

except for

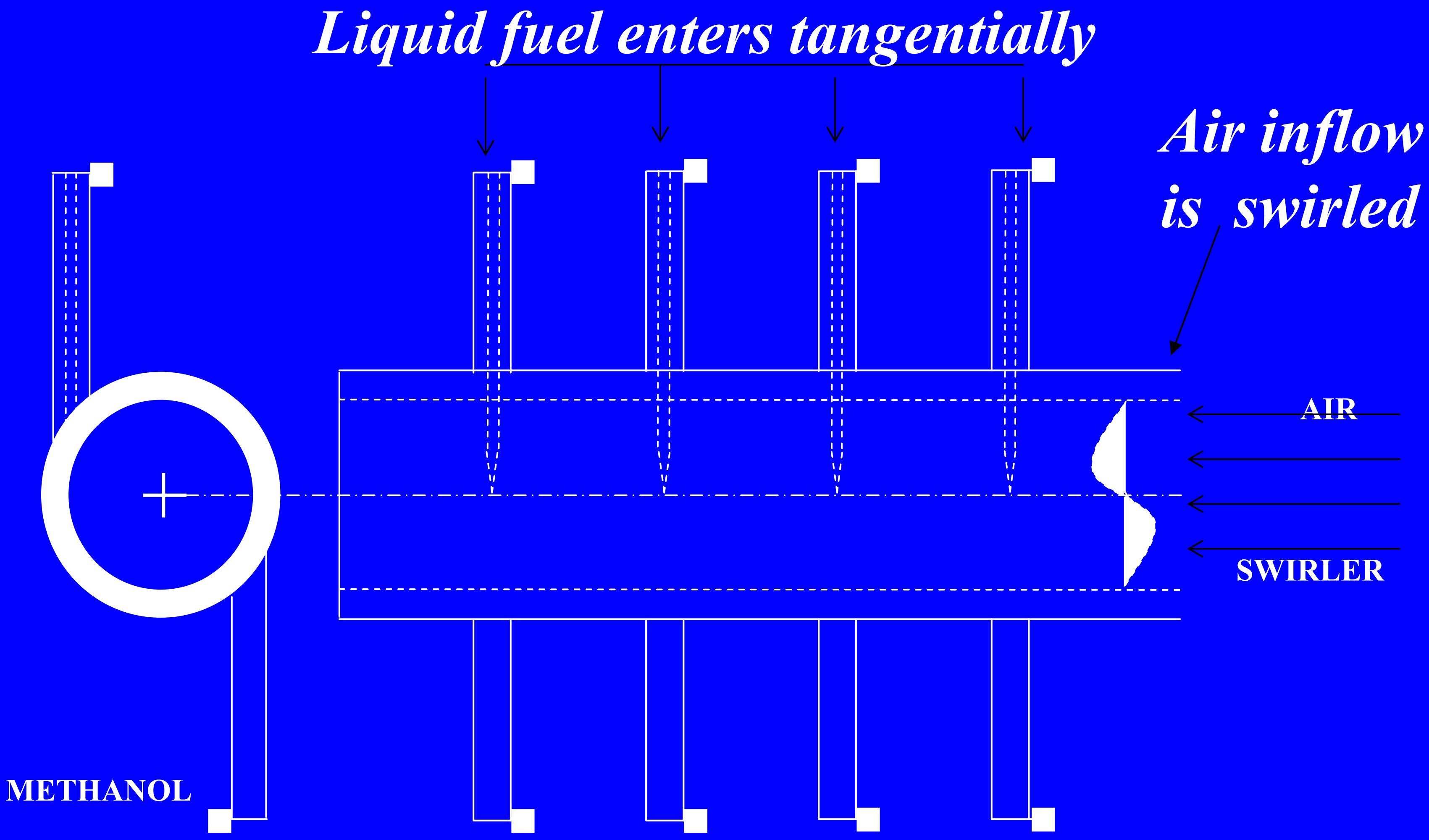
$$\delta_F = (1 - Y_F)\dot{M} = (1 - Y_F)\sum_k n^{(k)} \dot{m}^{(k)}$$

$$L(S) = -\frac{\dot{M}}{\theta}\left[S_v - S_{vs} \right]$$

The r-h-s source term is evaluated in the gas film around vaporizing droplets.

A uniform value of S can be obtained; S_v in regions without oxidizer equals S in regions with oxidizer.

Liquid Film Combustor


Liquid fuel enters tangentially

Air inflow is swirled

A thin film of fuel vaporizes from the wall

Liquid- Film Burning

Consider axisymmetric steady flow in cylindrical geometry. Neglect radial Stefan flow. Diffusion flame temperature and liquid–surface temperature can be determined by the same approach as for an isolated droplet.

$$S=h+\nu\left(Q-L_{eff}\right)Y_O+L_{eff}Y_F$$

$$h\left(T_f\right)=h\left(T_s\right)+L_{eff}Y_{Fs}\left(T_s\right)$$

$$\frac{h+\nu QY_O-h_s}{h_\infty+\nu QY_{O\infty}-h_s}=\frac{\nu Y_O-Y_F+Y_{Fs}}{\nu Y_{O\infty}+Y_{Fs}}$$

With Oseen approximation, S-Z variables are related to Bessel functions.

$$=\sum_{n=1}^{\infty}\frac{2}{\eta_n J_1(\eta_n)}J_0(\eta_n r/R_i)e^{-(\eta_n^2 x/PeR_i)}$$

$$J_0(\eta_n)=0$$